In a perfectly functioning world, every piece of information should be reflected simultaneously in the underlying spot market and its futures markets. However, in reality, information can be disseminated in one market first and then transmitted to other markets due to market imperfections. And, if one market reacts faster to information than the other, a lead-lag relation is observed. The lead-lag relationship in returns and volatilities between spot and futures markets is of interest to academics, practitioners, and regulators. In India, there are very few studies which have investigated the lead-lag relationship in the first moment of the spot and futures markets. This study investigates the lead-lag relationship in the first moment as well as the second moment between the S&P CNX Nifty and the Nifty future. It also investigates how much of the volatility in one market can be explained by volatility innovations in the other market and how fast these movements transfer between these markets.

- It conducts Multivariate Cointegration tests on the long-run relation between these two markets.
- It investigates the daily price discovery process by exploring the common stochastic trend between the S&P CNX Nifty and the Nifty future based on vector error correction model (VECM).
- It examines the volatility spillover mechanism with a bivariate BEKK model.
- Finally, this study captures the effects of recent policy changes in the Indian stock market.

The results reveal the following:

- The VECM results show that the Nifty futures dominate the cash market in price discovery.
- The bivariate BEKK model shows that although the persistent volatility spills over from one market to another market bi-directionally, past innovations originating in future market have the unidirectional significant effect on the present volatility of the spot market.

The findings of the study thus suggest that the Nifty future is more informationally efficient than the underlying spot market. These findings may provide insights on the information transaction and index arbitrage between the CNX Nifty and futures markets.
In perfectly efficient futures and spot markets, informed investors are indifferent between trading in either market, and new information is reflected in both, simultaneously. However, if one market reacts faster to information, and the other market is slow to react, due to market frictions such as transactions costs or market microstructure effects, a lead-lag relation in returns is observed. The market that provides greater liquidity, lower transaction costs, and less restriction, is likely to play a more important role in price discovery. Apart from information contained in prices themselves, volatility is also an important source of information. Ross (1989) demonstrated that the rate of information transmission is critically linked to the volatility. Therefore, the study of volatility spillover, the process by which the volatility in one market affects that of another market, furthers the understanding of information transmission in financial markets.

Thus, the lead-lag relationship in returns and volatilities between spot and futures markets is of interest to academics, practitioners, and regulators for a variety of reasons. Firstly, the issue is linked to market efficiency and arbitrage. Secondly, it is believed that futures markets potentially provide an important function of price discovery. If so, then futures prices should contain useful information about subsequent spot prices, beyond that already embedded in the current spot price. Thirdly, if volatility spillovers exist from one market to the other, then the volatility transmitting market may be used by market agents, which need to cover the risk exposure that they face, as a vehicle of price discovery. For example, the instantaneous impact and lagged effects of shocks between spot and futures prices is of interest, since such information may be used in decision making regarding hedging activities (Wahab and Lashgari, 1993). Thus, a better understanding of the dynamic relation of spot and futures prices and its relation to the basis provides to these “agents” the ability to use hedging in a more efficient way. Furthermore, if a return analysis is inconclusive, volatility spillovers provide an alternative measure of information transmission (Chan, Chan and Karolyi, 1991).

For all these reasons, research devoted towards the relationship between futures and spot returns (first moments) has been voluminous (see Chan, Chan and Karolyi, 1991: Chan, 1992, amongst others), with this interest expanding to examining higher moment dependencies (time-varying spillovers) between markets (see Ng and Pirrong, 1996; and Koutmos and Tucker, 1996, among others).

In the spot index futures market, Kawaller, Koch and Koch (1987) put forward the general principle that spot prices are affected by their past history, current and past futures prices, and other market information. Likewise, futures prices are affected by their past history, current and past spot prices, and other market information. Thus, causality is likely to be bi-directional. Ng (1987) finds that futures returns generally lead spot returns for a variety of futures contracts, including the S&P 500 Index. Herbst, McCormack and West (1987) use tick-by-tick data to demonstrate a lead-lag relationship between the S&P 500 Index futures, the Kansas City Board of Trade Value Line Index futures, and their respective spot indices. Kawaller, Koch and Koch (1987) show that, although S&P 500 cash returns sometimes lead futures for approximately one minute, the S&P 500 futures returns lead spot market returns by 20 to 45 minutes. Stoll and Whaley (1990) report similar results for the Major Market Index (MMI) and the S&P 500 Index. Chan (1992) and Ghosh (1993) further report the dominant role of S&P 500 futures in the price discovery process. However, using a cointegration approach like that of Ghosh, the study by Wahab and Lashgari (1993) finds that error-correcting price adjustments occur significantly in both the S&P 500 futures and cash markets in price discovery. The threshold error correction models of Dwyer, Locke and Yu (1996) and Martens, Kofman and Vorst (1998) (whereby the adjustment will only occur when deviations from equilibrium are larger than some threshold values) suggest that the S&P 500 futures market impounds information faster than the stock market. Wang and Wang (2001) examine the spot and forward exchange rates of the British, German, French, and Canadian currencies against the US dollar and conclude that there is a price discovery in both markets, implying a feedback effect between each pair of markets.

Emphasizing that volatility is a proxy for information flow, Chan, Chan and Karolyi (1991) examine the intraday volatility spillovers between the S&P 500 Index stock and futures markets by utilizing the generalized auto regressive conditional heteroskedasticity (GARCH) models. They show a strong cross-market dependence in the volatility process. In particular, innovations in either market will spill over to the other,
suggesting significant informational roles for both spot and futures markets. In contrast to Chan, Chan and Karolyi (1991), Koutmos and Tucker (1996) show that volatility spillovers only run from the futures market to the stock market; that is, innovations originating in the stock market have no impact on the futures market. Arshanapali and Doukas (1994) examine whether the S&P 500 index futures and the underlying spot index have the same volatility process. They report evidence against interdependence of volatilities in futures and spot markets. In the currency futures market, Chatrath and Song (1998) examine volatility spillover relationships between the spot and futures markets for Japanese Yen vs. the US Dollar and argue that the futures volatility influence the spot due to faster incorporation of market-related information, such as macroeconomic announcements in the US.

Most of the above studies have been conducted in developed countries more particularly in the US. In India there are very few studies which have investigated the lead-lag relationship in the first moment of the spot and futures markets. For example, Thenmozhi (2002); Anandbabu (2003) (Please give all the names in the Reference) etc., have found that the futures market in India has more power in disseminating information and therefore has been found to play the leading role in the matter of price discovery. Mukherjee and Mishra (2004) have investigated the possible lead-lag relationship, both in terms of return and volatility, among the Nifty spot index and index futures markets in India. Their results suggest that there is a strong contemporaneous and bidirectional relationship between the returns in the spot and futures markets.

The current study investigates the lead-lag relationship in the first moment as well as the second moment between the S&P CNX Nifty and the Nifty future. This study differs from the previous studies in India in several aspects. First, it conducts Multivariate cointegration tests on the long-run relation between these two markets. Second, it investigates the daily price discovery process by exploring the common stochastic trend between the S&P CNX Nifty and the Nifty future based on vector error correction model (VECM). Thirdly, it examines the volatility spillover mechanism with a bivariate BEKK model. Finally, this study uses more current data to capture the effects of recent policy changes in the Indian stock market.

The VECM results show that the Nifty futures dominate the cash market in price discovery. The bivariate BEKK model shows that although the persistent volatility spills over from one market to another market bi-directionally, past innovations originating in future market have the unidirectional significant effect on the present volatility of the spot market. The findings of the study may provide insights on the information transaction and index arbitrage between the CNX Nifty and futures markets.

METHODOLOGY

Cointegration and Price Discovery

Regressing non-stationary variables on each other leads to potentially misleading inferences about the estimated parameters and the degree of association. Therefore, before testing for cointegration, the order of integration of price series must be determined. To identify whether our series are $I(1)$, we employ both the augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1979) and the Phillips-Perron (P-P) test (1988).

\[ \Delta X_t = \rho_0 + \rho X_{t-1} + \sum_{i=1}^{n} \delta_i \Delta X_{t-i} + \epsilon_t \]  

where $X_t$ is the log price series, $\rho_0$ is a constant or drift, $\rho$ is $\alpha - 1$, $\Delta$ is the first difference operator, $\epsilon_t$ is a pure white noise error term and $\Delta X_{t-i} = (X_{t-i} - X_{t-i-1})$, $\Delta X_{t-i} = \Delta X_{t-i} - \Delta X_{t-i^{-1}}$, etc., $i = 1$ to $n$ is number of lagged difference terms which is determined empirically to remove any autocorrelation in error term $\epsilon_t$. The null hypothesis is to test that $\rho = 0$. If $\rho = 0$, then $\alpha = 1$, that is, we have a unit root, meaning the time series under consideration is nonstationary. But for stationarity, $\alpha$ must be less than one and hence $\rho$ must be negative.

\[ X_t = \alpha_0 + \alpha X_{t-1} + u_t \]  

where, $u_t$ is the error term. To test for a unit root, the regression $t$-statistic for the null hypothesis ($H_0: \alpha = 1$), denoted by $t_{\alpha}$, is adjusted nonparametrically to account for possible serial correlation in $u_t$.

\[ t_{\alpha} = \frac{\hat{\alpha} - 1}{\hat{\sigma}_{\alpha}} \]  

(a) Phillips-Perron (PP) regression:

Consider the following regression model for a time series ($X_t$):

\[ X_t = \alpha_0 + \alpha X_{t-1} + u_t \]  

(a) Augmented Dickey-Fuller regression:
If each price series is an I(1) process, the series can be modeled by cointegration analysis.

The concept of cointegration was first introduced by Granger (1981). Engle and Granger (1987) propose a procedure for testing the cointegration hypothesis. A level regression is performed to generate residuals which may be thought of as equilibrium pricing errors. Residuals are then subjected to tests for cointegration. The ADF test for cointegration is used for this purpose. With two time series—spot index \( S_t \) and Index futures prices \( F_t \) each of which is I(1), the cointegration regression equation is

\[
S_t = \eta_0 + \eta_1 F_t + s_t \tag{3}
\]

where, \( S_t \) is regressed on a constant \( \eta_0 \) and \( F_t \). \( \eta_1 \) is the regression coefficient and \( s_t \) is the residuals or error terms. Now, the spot index and index futures prices will be cointegrated if and only if \( s_t \) is stationary.

Tests for cointegration proposed by Engle and Granger (1987) rely on a superconvergence result and apply an OLS estimation to obtain parameter estimates of the cointegrating vector. Johansen (1988) and Johansen and Juselius (1990) derive maximum likelihood estimators of the cointegrating vectors for an autoregressive process with independent Gaussian errors and a likelihood ratio test for the number of cointegrating vectors. Their procedure has the advantage of taking into account the error structure of the underlying process. It enables us to estimate and test the equilibrium relationship among non-stationary series while abstracting from short-term deviations from equilibrium. Thus, it provides relatively powerful tests when the model is correctly specified.

Given a set of two I(1) series, Johansen (1988, 1991) tests are used to determine whether the series stand in a long-run relationship between them; that is, they are cointegrated. The following VECM (Johansen, 1988) is estimated:

\[
\Delta X_t = \Pi X_{t-1} + \epsilon_t / \Omega \sim (0, H) \tag{4}
\]

where \( X_t \) is an \( n \times 1 \) vector \((S_t, F_t)'\) of the spot index and index futures prices respectively, \( \Delta \) denotes the first difference operator, \( \epsilon_t \) is a \( n \times 1 \) vector of residuals \((\epsilon_{1t}, \epsilon_{2t})'\) that follow an as-yet-unspecified conditional distributed with mean zero and time-varying covariance matrix, \( H \). The VECM specification contains information on both the short- and long-run adjustment to changes in \( X_t \), via the estimated parameters \( \Pi \) and \( H \), which are defined as \(-I - \Pi_1 - \Pi_2 - \cdots - \Pi_r \) and \(-I - \Pi_1 + \Pi_2 - \cdots + \Pi_r \), respectively. \( \Pi \) through \( \Pi_r \) are \( 2 \times 2 \) matrices of coefficients. The \( \Pi \) term determines whether and to what extent the system of equation is cointegrated, and is known as the cointegrating constant in the VAR system.

Johansen and Juselius (1990) show that the coefficient matrix \( \Pi \) contains the essential information about the relationship between \( S_t \) and \( F_t \). Specifically, if rank \( (\Pi) = 0 \), then \( \Pi \) is \( 2 \times 2 \) zero matrix implying that there is no cointegration relationship between \( S_t \) and \( F_t \).

In this case, the VECM reduces to a Vector Autoregressive (VAR) model in first differences. If \( \Pi \) has a full rank, that is rank \( (\Pi) = 2 \), then all variables in \( X_t \) are I(0) and the appropriate modeling strategy is to estimate a VAR model in levels. If \( \Pi \) has a reduced rank, that is rank \( (\Pi) = 1 \), then there is a single cointegrating relationship between \( S_t \) and \( F_t \), which is given by any row of matrix \( \Pi \) and the expression \((\Pi)_{1,i} \), is the error correction term. In this case, \( \Pi \) can be factored into two separate matrices \( \alpha \) and \( \beta \), both of dimensions \( 2 \times 1 \), where \( 1 \) represents the rank of \( \Pi \), such as \( \Pi = \alpha \beta' \), where \( \beta' \) represents the vector of cointegrating parameters and \( \alpha \) is the vector of error correction coefficients measuring the speed of convergence to the long-run steady state.

Since rank \( (\Pi) \) equals the number of characteristic roots (or eigenvalues) which are different from zero, the number of distinct cointegrating vectors can be obtained by estimating the number of these eigenvalues, which are significantly different from zero. The characteristic roots of the \( n \times n \) matrix \( \Pi \), are the values of \( \lambda \) which satisfy the following equation \( |\Pi - \lambda I_n| = 0 \), where \( I_n \) is an \( n \times n \) identity matrix. Johansen (1988) proposes the following two statistics to test for the rank of \( \Pi \):

\[
\lambda_{\max}(r) = -T \sum_{i=r+1}^{n}(1-I) \tag{5}
\]

\[
\lambda_{\max}(r, r+1) = -T \sum_{i=r+1}^{n}(1-I) \tag{6}
\]

where, \( \lambda \) are the eigenvalues obtained from the estimate of the \( \Pi \) matrix and \( T \) is the number of usable observations. The \( \lambda_{\max} \) tests the null that there are at most \( r \) cointegrating vectors, against the alternative that the
number of cointegrating vectors is greater than \( r \) and the \( \lambda_{\text{max}} \) tests the null that the number of cointegrating vectors is \( r \), against the alternative of \( r+1 \).

If spot and future prices are cointegrated, then causality must exist in at least one direction (Granger, 1981). Granger causality can identify whether two variables move one after the other or contemporaneously. When they move contemporaneously, one provides no information for characterizing the other. If “\( S \) causes \( F \)”, then changes in \( S \) should precede changes in \( F \). Consider the VECM specification of Equation (4), which can be written as follows:

\[
\Delta S_t = a_S z_{t+1} + \sum_{i=1}^{p} b_{si} \Delta S_{t-i} + \sum_{i=1}^{p} c_{si} \Delta F_{t-i} + \epsilon_{s,t}
\]

\[
\Delta F_t = a_F z_{t+1} + \sum_{i=1}^{p} b_{fi} \Delta S_{t-i} + \sum_{i=1}^{p} c_{fi} \Delta F_{t-i} + \epsilon_{f,t}
\]

where \( \Delta S \) is the return series from spot market and \( \Delta F \) is the return series of futures market, \( b_{si}, c_{si} \), and \( c_{fi} \) are the short-run coefficients, \( z_{t+1} \) is the error correction term (ECT), and \( \epsilon_{s,t} \) and \( \epsilon_{f,t} \) are residuals (as explained earlier). The magnitude of the coefficients \( a_s \) and \( a_f \) determines the speed of adjustment back to the long-run equilibrium following a market shock. When these coefficients are large, adjustment is quick, so \( z \) will be highly stationary and reversion to the long-run equilibrium \( E(z) = c \Delta E(F) \) will be rapid.\(^2\)

When \( S \) and \( F \) are cointegrated asset prices, the ECM will capture dynamic correlations and causalities between their returns. If the coefficient on the lagged \( F \) returns in the \( S \) equation are found to be significant, then turning points in \( F \) will lead turning points in \( S \); that is, \( F \) Granger causes \( S \). There must be causalities when a spread is mean-reverting and two asset prices are moving in line, but the direction of causality may change over time.

**Volatility Spillovers**

Substantial attention has been focused on how news from one market affects the volatility process of another market. See, for instance, Hamao, Masulis and Ng (1990), Koutrmos and Booth (1995), and Lin, Engle and Ito (1994) in the US, UK, and Japanese stock markets; Booth, Martikainen and Tse (1996) (check year) and Christofi and Pericli (1999) in other international stock markets. All these articles use GARCH-type models to examine the volatility spillovers between markets. The theory of volatility spillovers based on the GARCH models is first introduced and named “meteor showers” by Engle, Ito and Lin (1990). Chan, Chan and Karolyi (1991) provide a detailed discussion on the need to focus on the volatility spillovers between the stock and futures markets. In particular, following Ross (1989), Chan, Chan and Karolyi (1991) contend that “it is the volatility of an asset’s price, and not the asset’s simple price change, that is related to the rate of flow of information to the market.”

Using a bivariate GARCH model (Bollerslev, 1986), we examine the patterns of information flows between two markets. Compared to a single-series univariate GARCH model, a bivariate model utilizes information in two different markets’ histories and examine the volatility spillover between two related markets. The bivariate modeling approach is well-suited for the analysis of the return series of spot and future markets, particularly because these two are shown to be cointegrated. The specific model we would use here is the BEKK model, after Baba, Engle, Kraft and Kroner who wrote the preliminary version of Engle and Kroner (1995). In this model, the variance-covariance matrix of equations depends on the squares and cross-products of innovation \( \varepsilon \) and volatility \( H_t \) for each market lagged one period. One important feature of this specification is that it builds in sufficient generality, allowing the conditional variances and covariances of the stock markets to influence each other, and, at the same time, does not require the estimation of a large number of parameters (Karolyi, 1995). The model also ensures the condition of a positive semi-definite conditional variance-covariance matrix in the optimization process, and is a necessary condition for the estimated variances to be zero or positive. Since the ARCH/GARCH methodology is, by now, well known, only a brief description of a bivariate BEKK model is provided in the text below.\(^3\)

The following mean equation is estimated for each return series given as:

\[
R_{ij} = \mu_i + \alpha R_{ij,t-1} + \varepsilon_{ij}
\]

where, \( R_{ij} \) is the continuously compounded return series on index \( i \) between time \( t-1 \) and \( t \), \( \mu_i \) is a long-term drift coefficient and \( \varepsilon_{ij} \) is the error term for the return on index \( i \) at time \( t \). Since we are interested in the possibility of volatility transmission between spot and futures markets, as well as persistence of volatility within each
market, we employ the bivariate BEKK model. The BEKK parameterization for the multivariate GARCH model is written as:

\[ H_t = C'A + \sum_{i=1}^{\infty} A_i' (\varepsilon_t, \varepsilon_t') A_i + \sum_{j=1}^{\infty} B_j' H_j B_j, \]

where, C, A and B are n x n matrices and C is a lower triangular.

This can be expressed for the bivariate case of the BEKK as:

\[ \begin{bmatrix} H_{11t} & H_{12t} \\ H_{21t} & H_{22t} \end{bmatrix} = C' C + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{11t} & \varepsilon_{12t} \\ \varepsilon_{21t} & \varepsilon_{22t} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} H_{11t-1} & H_{12t-1} \\ H_{21t-1} & H_{22t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \]

Where, \( H_{11t} \) and \( H_{22t} \) are the conditional variances of the first and second series, respectively and \( H_{12t} \) and \( H_{21t} \) are the conditional covariance between the two series. The \( C \), \( A \) and \( B \) are elements of a 2 x 2 symmetric matrices of elements \( \varepsilon_t \), the elements \( a_{ij} \) of the symmetric 2 x 2 matrix \( A \) measure the degree of innovation from market \( i \) to market \( j \). The elements \( b_{ij} \) of the symmetric 2 x 2 matrix \( B \) measure the degree of innovation from spot to futures and futures to spot markets, respectively. The elements \( \varepsilon_t \) of the symmetric 2 x 2 matrix \( C \) measure the persistence in conditional volatility between spot and futures markets. Alternatively, the elements \( a_{ij} \) and \( b_{ij} \) in conditional variance equation (11) measures the volatility spillover from spot to futures and futures to spot, respectively.

With the assumption that the random errors are normally distributed, the log-likelihood function for the multivariate GARCH model is:

\[ L(\theta) = -\frac{T}{2} \ln (2\pi) - \frac{1}{2} \sum_{t=1}^{T} \left( I_t' H_t^{-1} I_t + \varepsilon_t' \varepsilon_t / H_t \right) \]

where, \( T \) is the number of observations, \( \theta \) is the vector of parameters to be estimated, and all other variables are as previously defined. The BHHH (Berndt, Hall, Hall and Hausman) algorithm is used to produce the maximum likelihood parameter estimates and their corresponding asymptotic standard errors.

Lastly, the Ljung-Box Q statistic is used to test for randomness in the noise terms, \( \varepsilon_t \), for the estimated MGARCH model. This statistic is given by:

\[ Q = T(T+2) \sum_{j=1}^{p-k} (T-j)^{-1} r(j) \]

where, \( T \) is the number of observations, \( p \) is the lag length, \( r(j) \) is the sample autocorrelation at lag \( j \) calculated from the noise terms. \( Q \) is asymptotically distributed as \( \chi^2 \) with \( (p-k) \) degrees of freedom and \( k \) is the number of explanatory variables. The test statistic in (13) is used to test the null hypothesis that the model is correctly specified, or equivalently, that the noise terms are random.

**EMPIRICAL RESULTS**

**Data and Descriptive Statistics**

The daily S&P CNX Nifty spot and the Nifty futures data used in this study are obtained from NSE India website. The data span over a period of almost 7 years 9 months from June 12, 2000 (when the futures trading on index first started in NSE) to March 29, 2007. The S&P CNX Nifty futures contracts have a maximum of 3-month trading cycle — the near month (one), the next month (two), and the far month (three). A new contract is introduced on the trading day following the expiry of the near month contract. The new contract will be introduced for a three-month duration. This way, at any point in time, there will be 3 contracts available for trading in the market, i.e., one near month, one mid month and one far month duration respectively. The daily data on near month contract are used to represent the Nifty future price for the whole period. The well-known rationale behind the use of nearby contract is that it is the most actively traded contract. The more actively traded an instrument is, more is the information contained in its price. A nearby contract always exhibits more trading volume than those contracts of other maturities. A contract becomes “nearby” at the beginning of the previous contract’s expiration month, thereby avoiding the often-noted expiration effect.

Table 1 reports summary statistics for log price and return series under consideration. As expected, both log spot and log future series have similar mean, standard deviation, skewness, and kurtosis measures. The mean and standard deviation measures are also the same for two return series. However, the kurtosis and skewness (absolute) measures of the future return series are greater than that of the spot returns suggesting that the futures market may be more volatile than the spot and that these
series may not be normally distributed (more so for the futures series than the stock index series). Investors appear to get better risk return trade-offs in the spot market (std. deviation= 0.01558 for mean return = 0.0005) than the futures market (std. deviation=0.01649 for mean return = 0.0005). The Ljung-Box statistics provide evidence in favour of intertemporal dependencies, which may also be a possible sign of a non-normal distribution.

Stationarity and Multivariate Cointegration Tests

Given the time series nature of the data, an initial step in the analysis is to test whether each price or log price series is integrated [I(1)] or stationary [I(0)]. An I(1) time series is said to have a unit root and any shock to the series is permanent. To identify whether our series are I(1), we conduct ADF unit root tests and P-P tests as described in the Methodology Section.

Unit root tests are also presented in Table 1. The ADF and PP test statistics are shown for the log price series and the return series for both the spot and futures indices. Based on Schwarz information criteria, the optimal lag length chosen for ADF test is 2 for log price series and 1 for return series. As expected, both the ADF and PP test statistics indicate that the log price series contain a single unit root at 1% level of significance, implying the fact that both the log prices series are non-stationary. Both these test statistics, however, reject the hypothesis of a unit root at 1% level of significance in return series, implying the fact that the return series are stationary. The Johansen multivariate cointegration test results are reported in Table 2. Both the trace and maximal eigenvalue tests indicate that Nifty spot and futures markets are cointegrated with one cointegrating vector. In other words, the two markets share a long-run price equilibrium and are affected by one common stochastic factor.

Price Discovery

Table 3 displays the estimates of the speed of adjustment coefficients obtained by VECM estimation using equations (7) and (8). Based on the Akaike and Schwarz information criterion, the optimal lag length chosen for VECM estimation has turned out to be 3. The error correction (EC) term (\(a_i\)), which is also called the speed of adjustment coefficient, is significant in both equations with correct signs, suggesting a bidirectional error correction. However, the EC term in the spot equation (or the equation with \(\Delta S\), being the dependent variable, i.e., Equation 7) is greater in absolute term than that of the futures equation \([a_s = -0.328459 (t-stat. = -6.44568) vs. a_f = 0.179618 (t-stat. = 2.91583)]\). These results indicate that when the cointegrated series is in disequilibrium in the short run, it is the spot price that makes greater adjustment in order to reestablish the equilibrium. In other

<table>
<thead>
<tr>
<th>Table 1: Descriptive Statistics and Tests for Stationarity</th>
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<tbody>
<tr>
<td>Log of Spot Price</td>
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<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>Sample mean</td>
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<tr>
<td>St. Deviation</td>
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<tr>
<td>Skewness</td>
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<td>Kurtosis</td>
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<td>Maximum</td>
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<tr>
<td>Minimum</td>
</tr>
<tr>
<td>L-J Box</td>
</tr>
<tr>
<td>ADF t-statistic</td>
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<tr>
<td>PP t-statistic</td>
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*Returns are continuously compounded and multiplied by 100

<table>
<thead>
<tr>
<th>Table 2: Johansen Multivariate Cointegration Tests</th>
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<tbody>
<tr>
<td>Trace Test</td>
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<tr>
<td>------------</td>
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<tr>
<td>(\Pi = 0)</td>
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<tr>
<td>(\Pi = 1)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Max-Eigen Test</th>
<th>Max-Eigen Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Pi = 0)</td>
<td>521.8819</td>
<td>14.2646</td>
<td>0.0001</td>
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<tr>
<td>(\Pi = 1)</td>
<td>0.011511</td>
<td>3.8414</td>
<td>0.9143</td>
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</tbody>
</table>
words, the future price leads the spot price in price discovery. The first two lags of index innovations $b_{i,t}$ ($t=0.05118$), and $b_{j,t}$ ($t=1.35058$) in equation (8) are insignificant indicating that the spot market does not lead the futures market for two days. The significant index innovation occurs only at lag 3 ($b_{j,t}$) ($t=1.68406$) at 1% level, suggesting that the spot market leads the futures market only by at most 3 days. On the contrary, the futures innovations show a much stronger tendency to lead with all the lagged futures market innovations ($c_{i,t}, c_{j,t}$, and $c_{3,t}$ in equation 7) whose t-statistics are significant at 1%. The test for Granger causality from futures to spot is an F-test for the joint significance of $c_{i,t}$, $c_{j,t}$ and $c_{3,t}$ in an OLS regression. Similarly, the test for Granger causality from spot to futures is an F-test for the joint significance of $b_{i,t}, b_{j,t}$ and $b_{3,t}$. The F-statistics for $\sum_{i=1}^{7}c_{i} = 79.07223$ and for $\sum_{i=1}^{7}b_{i} = 4.059432$. The 1% critical value of the $F_{1,30}$ distribution is approximately 3.78, and so there was significant causality from spot to futures markets but a very much more significant causality from futures to spot. Thus, the above results provide a convincing evidence that though there is bidirectional causality, futures prices are better predictors of the near spot prices.

**Volatility Spillovers**

The volatility equation of the fitted BEKK(1,1) model is

$$[H_{i,t}, H_{j,t}] = \begin{bmatrix} H_{i,t} \{ H_{j,t} \} \\ H_{i,t} \{ H_{j,t} \} \end{bmatrix} = 
\begin{bmatrix} 0.0029 \quad 0 \\ 0.0033 \quad 0.0019 \end{bmatrix} \begin{bmatrix} 0.0029 \quad 0 \\ 0.0033 \quad 0.0019 \end{bmatrix} \begin{bmatrix} \epsilon_{i,t}^2 \quad \epsilon_{j,t}^2 \quad \epsilon_{i,t}^2 \quad \epsilon_{j,t}^2 \end{bmatrix} + 
\begin{bmatrix} 0.338 \quad 0.037 \\ 0.079 \quad 0.378 \end{bmatrix} \begin{bmatrix} \epsilon_{i,t}^2 \epsilon_{j,t}^2 \end{bmatrix} + 
\begin{bmatrix} 0.338 \quad 0.037 \\ 0.079 \quad 0.378 \end{bmatrix} \begin{bmatrix} \epsilon_{i,t}^2 \epsilon_{j,t}^2 \end{bmatrix} \begin{bmatrix} \epsilon_{i,t}^2 \quad \epsilon_{j,t}^2 \quad \epsilon_{i,t}^2 \quad \epsilon_{j,t}^2 \end{bmatrix} + 
\begin{bmatrix} 0.338 \quad 0.037 \\ 0.079 \quad 0.378 \end{bmatrix} \begin{bmatrix} \epsilon_{i,t}^2 \epsilon_{j,t}^2 \end{bmatrix} \begin{bmatrix} \epsilon_{i,t}^2 \quad \epsilon_{j,t}^2 \quad \epsilon_{i,t}^2 \quad \epsilon_{j,t}^2 \end{bmatrix} \begin{bmatrix} \epsilon_{i,t}^2 \epsilon_{j,t}^2 \end{bmatrix}$$

(14)

The coefficients of importance in the bivariate BEKK(1,1) model are $a_{i}$ and $b_{j}$. The elements $a_{i}$ are called ARCH coefficients, i.e., the coefficients of square of residual terms ($\epsilon_{i,t}^2$ and $\epsilon_{j,t}^2$), and $b_{j}$ are called the GARCH coefficients, i.e., the coefficients of variance-covariance term $h_{i,t}$. As has been mentioned earlier, $a_{i}$ measures the innovation from market $i$ to market $j$. Now for $i=j$, the elements $a_{i}$ becomes $a_{1}$ or $a_{2}$. The coefficient $a_{1}$ quantifies the effect of its own lagged squared residual term $\epsilon_{i,t}^2$ on the present volatility of the spot market ($h_{i,t}$). Similarly, the coefficient $a_{2}$ quantifies the effect of its own lagged squared residual term $\epsilon_{j,t}^2$ on the present volatility of the future market ($h_{j,t}$). When $i \neq j$, the element $a_{i,j}$ becomes $a_{1,2}$ or $a_{2,1}$, and the coefficient $a_{i,2}$ measures the degree of innovation from spot to futures market and the coefficient $a_{2,1}$ from futures to spot market. The elements $a_{i,j}$ of the symmetric matrix $A$, in equation (14) indicates that the effects of its own lagged innovation in two markets are large ($a_{11} = 0.338$, and $a_{22} = 0.179618$).

**Table 3: Estimated Coefficients of VECM**

<table>
<thead>
<tr>
<th>The CNX Nifty Spot</th>
<th>The Nifty Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_3$</td>
<td>-0.328459 (0.05000) [-6.4456]</td>
</tr>
<tr>
<td>$b_{31}$</td>
<td>-0.240255 (0.05068) [-4.7408]</td>
</tr>
<tr>
<td>$b_{32}$</td>
<td>-0.221712 (0.04300) [-5.1564]</td>
</tr>
<tr>
<td>$b_{33}$</td>
<td>-0.052439 (0.03215) [-1.63112]</td>
</tr>
<tr>
<td>$c_{31}$</td>
<td>0.376471 (0.05000) [7.52667]</td>
</tr>
<tr>
<td>$c_{32}$</td>
<td>0.209846 (0.04432) [4.73450]</td>
</tr>
<tr>
<td>$c_{33}$</td>
<td>0.085788 (0.03603) [2.3808]</td>
</tr>
<tr>
<td>$\sum_{i=1}^{7} c_{i} (F \text{ stat})$</td>
<td>79.07223$^*$</td>
</tr>
</tbody>
</table>

$\Delta S_{i} = a_{i} z_{i,t} + \sum_{i=1}^{7} b_{i} \Delta S_{i,t} + \sum_{i=1}^{7} c_{i} \Delta F_{i,t} + \epsilon_{i,t}$

$\Delta F_{i} = a_{i} z_{i,t} + \sum_{i=1}^{7} b_{i} \Delta S_{i,t} + \sum_{i=1}^{7} c_{i} \Delta F_{i,t} + \epsilon_{i,t}$

*significant at 1%. The critical value of $F_{3,1708}$ at 1% level of significance is approximately 3.78.

The values given in first parenthesis are standard errors and in second parenthesis are t-statistics.
= 0.378) and significant at 1% level of confidence, indicating the presence of strong ARCH effects. The effect of own lagged innovation is higher for future market than the spot market \( (a_{22}>a_{11}) \). In terms of the cross-volatility effects, it is observed that the value of \( a_{21} \) is 0.798 (\( p \) value = 1.878e-013) which is significant. This indicates that past innovations in future market have the effect on present volatility of spot market. However, the past innovations in spot markets have no significant effect on futures market, since \( a_{12} \) is 0.037 whose \( p \) value (= 0.1276) is found to be insignificant.

Now, let us examine the elements \( b_{ij} \) which measure the persistent volatility spillover from \( i \) market to \( j \) market. When \( i=j \), the elements \( b_{ii} \) measure the effect of its own lagged volatility on the present volatility of the spot (for \( b_{11} \)) and on the present volatility of the futures (for \( b_{22} \)). When \( i \neq j \), the elements \( b_{ij} \) measure the effect of cross volatility, i.e., \( b_{21} \) measures the effect of volatility spillover from spot to futures and \( b_{12} \) measures the effect of volatility spillover from futures to spot markets. The elements \( b_{ii} \) of the symmetric matrix \( B \) in the equation (14) indicates that all of the estimated coefficients are significant. For both spot and futures markets, their own volatility persistence is very high \( (b_{11}=0.930 \text{ and } b_{22}=0.922) \). However, the persistent volatility spillover from spot to futures \( (b_{12}=-0.032) \) and from futures to spot \( (b_{21}=-0.034) \), though significant, is negative. The result suggests that a 1% increase in persistent futures volatility decreases spot return volatility by 0.034% and a 1% increase in persistent spot volatility decreases futures volatility by 0.032%. Here, the volatility spillover from futures to spot is slightly higher than that from spot to futures (though negatively). Although, a persistent volatility of one market spills over to another market bidirectionally, the volatility spillovers from spot to futures are weaker.

The results of the diagnostic check on the behaviour of standardized residuals and squared residuals show mixed evidence. The L-jung box statistic for squared standardized residuals \( [Q^2(12)=3.299 (0.9930)] \) for the spot indicates that there is no autocorrelation left in the series. However, the significant value of \( Q^2(12) \) for the futures \( [62.024 (0.000)] \) indicates that there is still autocorrelation left in the residual series. The L-jung Box tests are applied to each series separately, and they do not check the serial correlation of the cross moment. Hence these tests are not really multivariate tests. To produce the multivariate test of the squared standardized residuals, the multivariate extension of the univariate Ljung-Box test, i.e., a multivariate portmanteau test is also calculated which is found to be 104.4893 (0.000). The significant value of the multivariate test statistics suggests misspecification of the model and hence the current results should be explained with caution. Higher order BEKK models, for example, BEKK(1,2), BEKK(1,3), BEKK(2,1), etc., have been tried. Although the spillover results are similar, these higher order models do not improve the specification diagnosis. To improve the specification diagnosis, one can try for asymmetric bivariate GARCH model which is used to detect the potential asymmetry of stock market response to good and bad news. A bivariate EGARCH model is especially suitable for investigating the asymmetric impact of good and bad news on the volatility transmission between markets. Unfortunately, strict computing requirements and the non-availability of the required software has not permitted the application of this model. One should also note that although the GARCH-type models are popular in modeling the volatility process in financial series, many previous studies do not report satisfactory diagnostic results.

**Explanation of the Findings**

Both the VECM and BEKK models indicate that futures market is more informationally efficient than the underlying spot market. The VECM provides evidence to support the dominant role of Nifty future in price discovery. The BEKK model indicates that although persistent volatility spills over from one market to another market bidirectionally, past innovations originating in futures market have the unidirectional significant effect on the present volatility of the spot market. Thus, the findings of the study support the notion that information disseminates in the futures market before the spot market. But what might be the reasons for the same? Conventional wisdom amongst professional traders suggests that movements in the futures price should reflect the expected future movements in the underlying cash price. The futures price should quickly reflect all available information regarding events that may affect the underlying and respond quickly to new information. The spot index should respond in a similar fashion, but for the index to react to the new information completely, the underlying stocks must all be revalued, i.e., every con-
constituent stock must re-evaluate the new information and adjust accordingly. Because most stocks are not traded as frequently as futures, the index will respond to new information with a lag. Consider a situation where a positive information has just arrived in the market. Now a trader has two options: either to buy underlying stocks of the Nifty futures or to purchase the Nifty futures. In this scenario, the futures trade can be executed immediately compared to trading in actual underlying stocks. This happens because the futures trading needs little initial cash outlay, whereas a spot trading would require a greater up-front investment and a probable longer implementation time for stock selection and numerous underlying stock transactions. Trading futures also has the advantages of a highly liquid market, easily available short positions, low margins, leveraged positions, and rapid execution. This transaction preference for futures may explain why information disseminates in futures market first and subsequently in the spot market.

SUMMARY AND CONCLUSION

In a perfectly functioning world, every piece of information should be reflected simultaneously in the underlying spot market and its futures markets. However, in reality, information can be disseminated in one market first and then transmitted to other markets due to market imperfections. This study examines the price discovery process of the S&P CNX Nifty and the Nifty futures. The study also investigates how much of the volatility in one market can be explained by volatility innovations in the other market and how fast these movements transfer between these markets. The findings of the study suggest that the Nifty futures are more important indicators of stock movements. The VECM provides evidence to support the dominant role of Nifty futures in price discovery, i.e., future prices tend to discover new information more rapidly than spot prices. The bivariate BEKK model shows that although the persistent volatility spillovers take place from one market to another market bi-directionally, past innovations originating in the future market have the unidirectional significant effect on the present volatility of the spot market. The results are in accordance with the results in most futures markets (see Chan, Chan and Karolyi, 1991; Chan, 1992; and Kavussanos and Nomikos, 2001, among others).

In conclusion, the Nifty futures is more informationally efficient than the underlying stock market. Because all the Nifty 50 component stocks are actively traded, these results should not be induced by nonsynchronous trading. Possible reasons are the inherent leverage, low transaction costs, and the absence of short sale restrictions in the futures market. The results have practical implications for investors who wish to improve portfolio performance. Investors may use the futures market to discover the new equilibrium price, where the mean of this equilibrium price may be transmitted to the spot market. Greater efficiency of price discovery of futures market may help investors with more efficient strategies for hedging and speculating in futures. Moreover, a better understanding of the interconnectedness of these markets would be useful for those policy makers who coordinate the stability of financial markets.

END NOTES

1. The detailed analysis of the P-P test statistic is given in any standard econometric text book.
2. The brief description of VECM and Cointegration model are given in Annexure 1. The majority of the description has been taken from Zivot and Jiahui (2002).
3. The brief description of ARCH/GARCH models are given in Annexure 2, while the BEKK model has been explained in some details in Annexure 3, which has been borrowed mainly from Engle and Kroner (1995).

ANNEXURE 1: VAR MODELS AND COINTEGRATION

The Granger representation theorem links cointegration to error correction models. In a series of important papers, Johansen (1988; 1991) firmly roots cointegration and error correction models in a vector autoregression framework. This Annexure outlines Johansen’s approach to cointegrating modeling.

Consider the levels VAR (p) model for the (nx1) vector $X_t$.

$$X_t = \Pi_1 X_{t-1} + \ldots + \Pi_p X_{t-p} + \varepsilon_t, t=1, \ldots, T,$$  

(1)
The VAR($p$) model is stable if
\[ \text{Det} (I_n - \Pi_1 - \ldots - \Pi_p) = 0 \]  
(2)
has all roots outside the complex unit circle. If (2) has a root on the unit circle then some or all of the variables in $X_t$ are $I(1)$ and they may also be cointegrated. Now, $X_t$ is cointegrated if there exists some linear combination of the variables in $X_t$ that is $I(0)$.

Suppose $X_t$ is $I(1)$ and possibly cointegrated, then VAR representation (1) is not the most suitable representation for analysis because the cointegrating relations are not explicitly apparent. The cointegrating relations become apparent if the levels VAR (1) is transformed to the vector error correction model (VECM):
\[ \Delta X_t = \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_p \Delta X_{t-p+1} + \varepsilon_t \]  
(3)
where $\Pi = \Pi_1 + \ldots + \Pi_p - I_n$ and $\Gamma_j = - \sum_{j=k+1}^{p} \Pi_j$, $k=1, \ldots, p-1$. The matrix $\Pi$ is called the long-run impact matrix and $\Gamma_j$ are the short-run impact matrices. Notice that the VAR parameters $\Pi$ may be recovered from the VECM parameters $\Pi$ and $\Gamma_j$ via
\[ \Pi_i = \Gamma_i + \Pi_1 I_n \]  
(4)
\[ \Gamma_j = \Gamma_j I_n, j=2, \ldots, p. \]

In the VECM (3), $\Delta X_t$ and its lags are $I(0)$. The term $\Pi X_{t-1}$ is the only one which includes potential $I(1)$ variables and for $\Delta X_t$ to be $I(0)$, $\Pi X_{t-1}$ must also be $I(0)$. Therefore, $\Pi X_{t-1}$ must contain the cointegrating relations if they exist. If the VAR($p$) process has unit roots, then from (2) it is clear that $I$ is a singular matrix. If $\Pi$ is singular, then it has reduced rank; that is rank ($\Pi$) = $r < n$. There are two cases to consider:

1. **rank ($\Pi$) = 0.** This implies that $\Pi = 0$ and $X_t$ is $I(1)$ and not cointegrated. The VECM (3) reduces to a VAR (p-1) in first differences
\[ \Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_p \Delta X_{t-p+1} + \varepsilon_t \]

2. **$0 < \text{rank} (\Pi) = r < n$.** This implies that $X_t$ is $I(1)$ with $r$ linearly independent cointegrating vectors and $n-r$ common stochastic trends (unit roots). Since $\Pi$ has rank $r$, it can be written as the product
\[ \Pi = \alpha \beta' \]  
where $\alpha$ and $\beta$ are ($n \times r$) matrices with rank ($\alpha$) = rank ($\beta$) = $r$. The rows of $\beta$ form a basis for the $r$ cointegrating vectors and the elements of $\alpha$ distribute the impact of the cointegrating vectors to the evolution of $\Delta X_t$. The VECM (3) becomes
\[ \Delta X_t = \alpha \beta' X_{t-1} + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_p \Delta X_{t-p+1} + \varepsilon_t \]  
(5)
where $\beta' X_{t-1} \sim I(0)$ since $\beta'$ is a matrix of cointegrating vectors.

It is important to recognize that the factorization $\Pi = \alpha \beta'$ is not unique since for any $r \times r$ nonsingular matrix $H$ we have
\[ \alpha \beta' = \alpha H \beta' = (\alpha H) (\beta H')' = \alpha' \beta'. \]

Hence the factorization $\Pi = \alpha \beta'$ only identifies the space spanned by the cointegrating relations. To obtain unique values of $\alpha$ and $\beta'$ requires further restrictions on the model.

**A Bivariate Cointegrated VAR (1) Model**

Consider the bivariate VAR(1) model for $X_t = (x_{1t}, x_{2t})'$

\[ X_t = \Pi_1 X_{t-1} + \varepsilon_t \]

The VECM is
\[ \Delta X_t = \Pi X_{t-1} + \varepsilon_t \]

where $\Pi = \Pi_1 - I_2$. Assuming $X_t$ is cointegrated, there exists a 2x1 vector $\beta = (\beta_1, \beta_2)'$ such that $\beta' X_t = \beta_1 y_{1t} + \beta_2 y_{2t}$ is $I(0)$. Using the normalization $\beta_1 = 1$ and $\beta_2 = -\beta$ the cointegrating relation becomes $\beta' X_t = y_{1t} - \beta y_{2t}$. This normalization suggests the stochastic long-run equilibrium relation
\[ y_{1t} = \beta y_{2t} + u_t \]

where $u_t$ is $I(0)$ and represents the stochastic deviations from the long-run equilibrium $y_{1t} = \beta y_{2t}$.

Since $X_t$ is cointegrated with one cointegrating vector, rank ($\Pi$) = 1 and can be decomposed as
\[ \Pi = \alpha \beta' = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} (1 - \beta) = \begin{pmatrix} \alpha_1 - \alpha_2 \beta \\ \alpha_2 - \alpha_2 \beta \end{pmatrix} \]
The elements in the vector $\alpha$ are interpreted as speed of adjustment coefficients. The cointegrated VECM for $X_i$ may be rewritten as

$$\Delta X_i = \alpha_i + c_i.$$  

(6)

Writing the VECM equation by equation gives

$$\Delta x_{it} = \alpha_i (x_{it-1} - \beta x_{it-1}) + \epsilon_{it},$$

(7)

$$\Delta x_{it} = \alpha_i (x_{it-1} - \beta x_{it-1}) + \epsilon_{it},$$

(8)

The first equation relates the change in $y_{it}$ to the lagged disequilibrium error $\beta X_{it-1} = (x_{it-1} - \beta x_{it-1})$ and the second equation relates the change in $y_{it}$ to the lagged disequilibrium error as well. Notice that the reactions of $x_1$ and $x_2$ to the disequilibrium errors are captured by the adjustment coefficients $\alpha_1$ and $\alpha_2$.

Here one can see the relationship between error-correcting models and cointegrated variables. By assumption, $x_1$ is stationary, so that the left-hand side of (7) is $I(0)$. For (7) to be sensible, the right-hand side must be $I(0)$ as well. Given that $\epsilon_{it}$ is stationary, it follows that the linear combination $x_{it-1} - \beta x_{it-1}$ must also be stationary; hence, the two variables $X_1$ and $X_2$ must be cointegrated with the cointegrating vector $(1 - \beta)$.

The essential point to note is that the error-correction representation necessitates the two variables be cointegrated of order CI (1, 1). This result is unaltered if we formulate a more general model by introducing the lagged changes of each variable into both equations:

$$\Delta X_{it} = a_1 z_{it-1} + \sum_{i=1}^p b_{i1} \Delta X_{it-i} + \sum_{i=1}^q c_{i1} \Delta X_{it-i} + \epsilon_{it},$$

(9)

$$\Delta X_{it} = a_1 z_{it-1} + \sum_{i=1}^p b_{i2} \Delta X_{it-i} + \sum_{i=1}^q c_{i2} \Delta S_{it-i} + \epsilon_{it},$$

(10)

where $z_{it} = x_{it-1} - \beta x_{it-1}$. Again $\epsilon_{t1}, \epsilon_{t2}, \epsilon_{t3}$ and all terms involving $\Delta X_{it-1}$ and $\Delta X_{it-2}$ are stationary. Thus, the linear combination of variables $(x_{it-1} - \beta x_{it-1})$ may be stationary as well.

ANNEXURE 2: ARCH AND GARCH

Let $R_t$ be the rate of return of a particular stock or market portfolio from time $t-1$ to time $t$. Also let $\psi_t$ be the information set containing the realized values of all relevant variables up to time $t-1$. Since investors know the information in $\psi_t$, when they make their investment decision at time $t-1$, the relevant expected return and volatility to the investors are the conditional expected value of $R_t$, given $\psi_t$, and the conditional variance of $R_t$ given $\psi_t$. These are denoted by $m_t$ and $h_t$ respectively. That is, $m_t = E(R_t / \psi_t)$ and $h_t = Var(E(R_t / \psi_t))$. Given these definitions, the return series $R_t$ can be defined as

$$R_t = E(R_t / \psi_t) + \epsilon_t = m_t + \epsilon_t,$$

(11)

where the unexpected return at time $t$ is $\epsilon_t = R_t - m_t$. Here, $\epsilon_t$ is treated as a collective measure of news at times $t$. A positive $\epsilon_t$ (an unexpected increase in price) suggests the arrival of good news, while a negative $\epsilon_t$ (an unexpected decrease in price) suggests the arrival of bad news. Further, a large value of $|\epsilon_t|$ implies that the news is ‘significant’ or big in the sense that it produces a large unexpected change in price.

Engle (1982) suggests that the conditional variance $h_t$ can be modeled as a function of the lagged $\epsilon_t$’s. That is, the predictable volatility is dependent on past news. The most detailed model he develops is the $q$th order autoregressive conditional heteroskedasticity model, the ARCH(q):

$$h_t = \omega + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \ldots + \alpha_q \epsilon_{t-q}^2$$

(12)

where $\omega > 0$, $\alpha_1, \alpha_2, \ldots, \alpha_q \geq 0$ and $\epsilon_t / \psi_t \sim N(0, h_t)$. The effect of a return shock $i$ periods ago ($i \leq q$) on current volatility is governed by the parameter $\alpha_i$. Normally, we would expect that $\alpha_i < \alpha_j$ for $i > j$. That is, the older the news, the less effect it has on current volatility. In an ARCH(q) model, old news which arrived at the market more than $q$ periods ago has no effect at all on current volatility. Alternatively, if a major market movement occurred yesterday, the day before or up to $q$ days ago, the effect will be to increase today’s conditional variance.

Bollerslev (1986) generalizes the ARCH (q) model to the GARCH (p, q) model, such that

$$h_t = \omega + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \ldots + \alpha_q \epsilon_{t-q}^2 + \beta_1 h_{t-1} + \beta_2 h_{t-2} + \ldots + \beta_q h_{t-q}$$

(13)

where $\omega > 0$, $\alpha_1, \alpha_2, \ldots, \alpha_q, \beta_1, \beta_2, \ldots, \beta_q \geq 0$ and $h_t / \psi_t \sim N(0, h_t)$. The GARCH (p,q) process defined above is stationary when $(\alpha_1 + \alpha_2 + \ldots + \alpha_q) + (\beta_1 + \beta_2 + \ldots + \beta_q) < 1$. The simplest but often very useful GARCH process is the GARCH (1, 1) process which is also called the generic or ‘vanilla’ GARCH model given by

$$h_t = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}$$

(14)
where \( \omega > 0, \alpha_i \geq 0, \beta_i = 0 \). The stationary condition for GARCH (1, 1) is \( \alpha_1 + \beta_1 < 1 \).

In the GARCH (1, 1) model, the effect of a return shock on current volatility declines geometrically over time. As referred earlier the GARCH (1, 1) is found to be an excellent model for a wide range of financial data (Bollerslev et al. 1992).

ANNEXURE 3: MULTIVARIATE GARCH

The extension from a univariate GARCH model to an \( n \)-variate model requires allowing the conditional variance-covariance matrix of the \( n \)-dimensional zero mean random variables \( \varepsilon_t \) to depend on elements of the information set. Letting \( H_t \) be measurable with respect to \( \Psi_t \), the multivariate GARCH model can be written as

\[
\varepsilon_t / \Psi_t - 1 \sim N(0, H_t).
\]

The parameterization for \( H_t \), as a function of the information set \( \Psi_t \), chosen here allows each element of \( H_t \) to depend on \( q \) lagged values of the squares and cross-products of \( \varepsilon_t \), as well as \( p \) lagged values of the elements of \( H_t \), and a \( j \times 1 \) vector of weakly exogenous variables (as defined by Engle, Hendry and Richard, 1983), \( x_t \). So the elements of the covariance matrix follow a vector ARMAX process in squares and cross-products of the residuals. We will assume \( x_t \) contains only current and lagged exogenous variables. Defining

\[
h_t = \text{vec } H_t, \quad \tilde{x}_t = \text{vec}(x_t, x_t'), \quad \text{and } \eta_t = \text{vec}(\varepsilon_t, \varepsilon_t'),
\]

where vec(.) is the vector operator that stacks the columns of the matrix, a parameterization can be written as:

\[
h_t = C_0 + C_1 + A_1 \eta_t - 1 + \ldots + A_q \eta_t - q + G_1 h_t - 1 + \ldots + G_p h_t - p,
\]

(15)

where \( C_0 \) is an \( n^2 \times 1 \) parameter vector, \( C_1 \) is an \( n^2 \times j \) parameter matrix, and \( A_i \) and \( G_i \) are \( n^2 \times n^2 \) parameter matrices.

Now, consider a simple two-equation GARCH (1,1) vec model without exogenous influences. Equation (15) becomes

\[
h_t = h_{11,t} = c_{01} + a_{11} \varepsilon_{1,t-1}^2 + g_{11} h_{11,t-1}
\]

\[
h_{12,t} = c_{02} + a_{12} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + g_{12} h_{12,t-1}
\]

\[
h_{21,t} = c_{03} + a_{21} \varepsilon_{2,t-1} \varepsilon_{1,t-1} + g_{21} h_{21,t-1}
\]

\[
h_{22,t} = c_{04} + a_{22} \varepsilon_{2,t-1}^2 + g_{22} h_{22,t-1}
\]

Notice that we have omitted the equation for \( h_{22,t} \) and have given no coefficient to \( \varepsilon_{2,t-1} \varepsilon_{1,t-1} \) or \( h_{21,t} \), as these are clearly redundant, leaving nine free parameters in each of the \( A_i \) and \( G_i \) matrices. Similar redundancies appear in the general \( n \)-variate GARCH (1, 1) vec model. In particular, all the covariance equations appear twice (i.e., there is an equation for \( h_{ij,t} \) as well as for \( h_{ji,t} \)) and all the off-diagonal terms appear twice within each equation (i.e., both of the terms \( \varepsilon_{i,t-1} \varepsilon_{j,t-1} \) and \( \varepsilon_{j,t-1} \varepsilon_{i,t-1} \) and both of the terms \( h_{ij,t} - 1 \) and \( h_{ji,t} - 1 \) appear in each equation). The redundant terms can be eliminated without affecting the model, leaving a total of \((n(n+1))/2\) unique parameters in each of the \( A_i \) and \( G_i \) matrices. In a direct formulation of (15), there appears to be \( n^4 \) parameters in each matrix, but many of these are superfluous.

For empirical implementation, it is desirable to restrict further this parameterization. A natural restriction that was first used in the ARCH context by Engle, Granger, and Kraft (1984) and in the GARCH context by Bollerslev, Engle, and Wooldridge (1988) is the diagonal representation, in which each element of the covariance matrix, \( h_{jk,t} \), depends only on past values of itself and past values of \( \varepsilon_j, \varepsilon_k \). That is, variances depend solely on past own squared residuals, and covariances depend solely on past own cross-products of residuals. This seems an intuitively plausible restriction because information about variances is usually revealed in squared residuals, and if the variances are evolving slowly, then past squared residuals should be able to forecast future variances. A similar argument can be made for covariances. In the vec model, a diagonal representation is obtained if the matrices \( A_i \) and \( G_i \) are assumed to be diagonal.

To illustrate in the bivariate case, the diagonal model is simply

\[
h_t = \begin{bmatrix}
    c_{01} \\
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
    \varepsilon_{1,t-1}^2 \\
    \varepsilon_{1,t-1} \varepsilon_{2,t-1} \\
    \varepsilon_{2,t-1} \varepsilon_{1,t-1} \\
    \varepsilon_{2,t-1}^2
\end{bmatrix}
+ \begin{bmatrix}
    g_{11} \\
    g_{12} \\
    g_{21} \\
    g_{22} \\
    g_{31} \\
    g_{32} \\
    g_{33}
\end{bmatrix}
\begin{bmatrix}
    h_{11,t-1} \\
    h_{12,t-1} \\
    h_{21,t-1} \\
    h_{22,t-1}
\end{bmatrix}
\]

or

\[
h_{11,t} = c_{01} + a_{11} \varepsilon_{1,t-1}^2 + g_{11} h_{11,t-1}
\]

\[
h_{12,t} = c_{02} + a_{12} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + g_{12} h_{12,t-1}
\]

\[
h_{21,t} = c_{03} + a_{21} \varepsilon_{2,t-1} \varepsilon_{1,t-1} + g_{21} h_{21,t-1}
\]

\[
h_{22,t} = c_{04} + a_{22} \varepsilon_{2,t-1}^2 + g_{22} h_{22,t-1}
\]
\[ h_{i,i} = c_{i} + a_{i1} \epsilon_{1,t-1}^2 + a_{i2} \epsilon_{2,t-1}^2 + 2 a_{i3} \epsilon_{1,t-1} \epsilon_{2,t-1} + b_{i1} h_{1,1,t-1}^2 + b_{i2} h_{2,2,t-1}^2 \]

In the bivariate model illustrated here, there are three free parameters in each of the \( A_i \) and \( G_i \) matrices, and in the general \( n \)-variate diagonal model, there are \(((n^2 + n)/2)\) free parameters in each matrix.

For any parameterization to be sensible, we require that \( H \) be positive definite for all values of \( \epsilon_i \) and \( x_i \) in the sample space. In the vec representation, and even in the diagonal representation, this restriction can be difficult to check, let alone impose during estimation. Now, a new parameterization is proposed that easily imposes these restrictions and that eliminates very few if any interesting models allowed by the vec representation.

Consider the following model:

\[ H_t = C_t' C_t + A_i \varepsilon_{i,t-1} A_i' + B_j H_{t-1} B_j, \]

where, \( C_t, A_i, \) and \( G_i \) are \( n \times n \) parameter matrices with \( C_t \) triangular; \( C_t \) are \( j \times n \) parameter matrices; and the summation limit \( K \) determines the generality of the process. It should be clear that (16) will be positive definite under very weak conditions. Furthermore, this representation is sufficiently general that it includes all positive definite diagonal representations and nearly all positive definite vec representations. It will be shown to be a particularly convenient representation for estimation and for analysis of simultaneous equations systems. Throughout the paper, we will refer to this representation as the BEKK representation.

To illustrate the BEKK model, consider first the simple GARCH (1, 1) model, with \( K = 1 \) and no exogenous influences:

\[ H_t = C_t' C_t + A_i (\varepsilon_{i,t-1} \varepsilon_{i,t-1}') A_i' + B_j H_{t-1} B_j. \]

In the bivariate case, which is illustrated for both the vec and diagonal representations earlier, the BEKK model becomes

\[
\begin{pmatrix}
H_{11,t} & H_{12,t} \\
H_{21,t} & H_{22,t}
\end{pmatrix}
= C_t' C_t +
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}'
\begin{bmatrix}
\varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1} \varepsilon_{2,t-1} \\
\varepsilon_{2,t-1} \varepsilon_{1,t-1} & \varepsilon_{2,t-1}^2
\end{bmatrix}
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\]

\[+
\begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix}'
\begin{bmatrix}
H_{11,t-1} & H_{12,t-1} \\
H_{21,t-1} & H_{22,t-1}
\end{bmatrix}
\begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix}
\] (17)

where \( c_t \) are elements of a \( 2 \times 2 \) symmetric matrix of constants \( C \), the elements \( a_{i,j} \) of the symmetric \( 2 \times 2 \) matrix \( A \), measure the degree of innovation from index \( i \) to index \( j \). More particularly, the elements \( a_{i,j} \) for \( i \neq j \) measure the degree of innovation from \( i \) to \( j \) and from \( j \) to \( i \) index, respectively. The elements \( b_{i,j} \) of the symmetric \( 2 \times 2 \) matrix \( B \) indicate the persistence in conditional volatility between \( i \) index and \( j \) index. Alternatively, the elements \( b_{i,j} \) for \( i \neq j \) in conditional variance equation (17) measure the volatility spillover from \( i \) to \( j \) and from \( j \) to \( i \) index, respectively. The total number of estimated elements for the variance equations is \( n(5n+1)/2 \). In our bivariate case, the number of elements is \( 2(5n+1)/2 = 11 \).

The conditional variance for each equation, ignoring the constant terms, can be expanded for the bivariate GARCH (1,1) as:

\[ h_{1,i} = a_{11} \epsilon_{1,t}^2 + 2 a_{12} \epsilon_{1,t} \epsilon_{2,t} + a_{12} \epsilon_{2,t}^2 + b_{11} h_{1,1,t}^2 + b_{12} h_{2,2,t}^2 \]
\[ h_{2,i} = a_{21} \epsilon_{1,t}^2 + 2 a_{22} \epsilon_{1,t} \epsilon_{2,t} + a_{22} \epsilon_{2,t}^2 + b_{21} h_{1,1,t}^2 + b_{22} h_{2,2,t}^2 \]

(18) (19)

Comparing this model to the vec form of the model, we see that this model economizes on parameters by imposing restrictions both across and within equations. In fact, for \( n = 2 \) we see that this representation uses only eight parameters, compared to the 18 from the vec model (excluding constants).

REFERENCES


Journal of Futures Markets, 14(8), 915-925.


Madhusudan Karmakar is an Associate Professor in the Finance and Accounts Area at the Indian Institute of Management, Lucknow. He teaches Financial Management and Investment Analysis and Portfolio Management. He has done his M.Com, M. Phil. (Finance) and Ph. D. (Finance) from the University of North Bengal. His research interest is in the area of stock market. He has published one book titled, Bubble: A Study of Scam, Scandal and Corruption in Indian Stock Market. He has contributed a large number of research papers in the area of finance and stock markets in reputed national and international journals.

e-mail: madhu@iiml.ac.in

What can we gain by sailing to the moon if we are not able to cross the abyss that separates us from ourselves? This is the most important of all voyages of discovery, and without it, all the rest are not only useless, but disastrous.

— Thomas Merton