Spectrum Auction and Investment in Telecom Industry: A Suggested Policy

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Recently, Indian telecommunication industry has passed through an auction process of 3G spectrum, which has enabled the government to collect hefty license fees. The successful bidders are expected to borrow this bid amount from the commercial banks and repay from the revenue to be generated from the new services.

This paper looks at this issue theoretically. The telecommunication firms operate on the basis of acquisition of airwave space, which is licensed by the government. The usual policy practice is to distribute the available spectrum by an ascending auction. As per the economic theory, auction has two merits. Firstly, auction ensures that spectrum goes to the most efficient firm, which is by definition the firm that makes the maximum profit among all. This firm by virtue of their maximum profit bids the highest amount in the auction and gets the spectrum. Secondly, spectrum auction ensures revenue maximization of the government. This result of efficiency optimization however holds under the assumption of perfect information and no uncertainty. Uncertainty and irrationality like over-optimism can lead to problems like ‘winners’ curse’ or ‘broke winners’ under auction. Still auction is market-based and less controversial than discretionary distribution.

However, this paper argues that high amount of auction bid can result in several deficiencies in the telecom industry. An issue of debate is whether high auction bid will result in higher price for the services. Under the assumption of infinite time horizon, high license fee is a sunk cost and therefore does not raise price. But this paper shows that a high license fee can raise the lending bank’s interest charge for which not only price rises but the infrastructural investment in the telecom industry suffers. After the acquisition of spectrum through auction the industry needs infrastructural investments to roll out the quality services. Inadequate investment in infrastructural activities like procurement of quality equipments, erection of tower, etc., may result in lower quantum of services with degraded qualities. This is in fact a reality in India where not only rural telecom infrastructure is vastly inadequate, the urban telecom infrastructure is also under severe stress.

In this paper, using the tool of game theory we show how capacity and quantity choice game of firms ends up in high auction bid and sub-optimal investment on infrastructure. Then how can the social planners simultaneously achieve the twin objective of growth and revenue maximization? The paper shows that these objectives are realized if a part of the license fee is spent for the subsidization of infrastructural investment in the telecom industry.
Recently, Indian telecommunication industry has passed through a spectrum auction process. The auction has evinced keen interest among the telecom players and the government has managed to collect hefty license fees out of this auction of 3G spectrum. But acquisition of spectrum cannot be the end of the story. The ultimate objective is to develop a vibrant telecom sector that serves the purpose of spreading positive network effects throughout the economy. This needs addressing the issue of critical link between spectrum auction and development of telecom infrastructure. This paper makes an attempt towards that direction in a theoretical perspective and brings home the point that simple revenue maximization by the government through auction of spectrum may not be able to fulfill the purpose of social welfare maximization.

Investment and growth in telecom sector is a *sine qua non* for the development of modern day economy. The benefits of investment in telecom spread across different sectors of the economy and different sections of the society. An estimate shows that about two-thirds of the growth of the US economy in the 1990s is driven by innovations in information technology (Jorgenson, 2001). But the progress of information technology sector depends critically on the spread of telecom facilities without which information acquisition and dissemination is not possible. A vibrant telecom sector is thus vital to the short- and long-term success of an economy in its development efforts.

**DYNAMICS OF SPECTRUM AUCTION**

The telecom industry uses radio spectrum for transmission of voice data and image. As the mobile telephony arrives at different countries, the need for more air spaces is felt and the respective governments start auctioning the spectrum. Usually the practice of ascending auction is followed where the highest payer gets the license. The pattern of auction in this particular scenario is a common value auction where for true evaluation of the uncertain prospect, the participating agents depend on their own as well as the others’ signals.

Auctions are theoretically classified as one of two types: Private-value auction and common-value auction. In private-value auction, bidders know their own valuation of the item with certainty, but there is uncertainty regarding other bidders’ valuation. In common-value auction, the true value of the item is the same to everyone, but different bidders have different estimates about the underlying value. A well-known phenomenon observed in bidding behaviour under common-value auction is *winner’s curse*, where the winning bid is very likely to be an overbid, resulting in eventual loss for the winner. Capen, Clapp and Campbell (1971) showed that in the 1960s and 1970s, oil companies had suffered unexpectedly low rates of return on oil field leasing and suggested that this was an outcome of winner’s curse. According to them, these low rates of return resulted from the fact that winning bidders naively based their bids on the unconditional expected value of the item (their own estimates of value), but ignored the fact that the winner’s highest bid exceeded the true value. To explain the issue, let us suppose that auction of a good with uncertain value \( V \) takes place. There are \( n \) buyers where bidder \( i \) has a signal of \( s_i \) on \( V \). Let the true value of the item be \( T = \sum s_i/n \). Now, by definition, the highest \( s_i \) that wins the bid exceeds \( T \) and the winner is a loser by paying more than the average value.

**Winner’s Curse**

There are several instances in the history of spectrum auctioning phenomena that may be termed as *winner’s curse*. In the year 1996, the United States Federal Communications Commission conducted C-block radio frequency spectrum auction. The Congress, concerned over balancing of the budget, counted that amount as a source of income. But few of the winners of the C-block auction made their payments. Many of them declared bankruptcy, including Next Wave, Pocket Communications, General Wireless, and Airadigm Communications (Zheng, 2001). Similarly, in India, after the liberalization of the telecommunication sector, hitherto characterized as a public sector monopoly, the entry of private operators began with the auctioning of licenses for basic and cellular services by the Department of Telecom. The entire country was divided into roughly 20 circles, categorized as A, B or C depending upon their revenue potential. The auction arrangement gave predominance to the need to generate revenues (Malik, 2004). Only 15 per cent weight was given to the speed at which the network could be rolled. Subsequently, it appeared that amongst all the circle licensees, only one licensee could post revenue higher than the license fee. Thus by 1998, almost all had defaulted on their license fees.
Beauty Contest

So, we see that common-value auction is often characterized by winner’s curse, as there is likelihood of the telecom license fee being higher than the profitable level. It is because of this reason that an alternative scheme of spectrum allocation, known as beauty contest, is sometimes recommended. The history of spectrum allocation has the records of both auctions and beauty contests. While countries like USA, UK, Netherlands, and Denmark have opted for auctions, countries like Sweden, Finland, and Portugal have resorted to beauty contest policy for the latest 3G licensing.

Under the mode of operation of the beauty contest, the government sets some criteria of selection and invites applications. These applications are then sorted out on the basis of scores on those pre-set criteria and licenses are allocated to those who earn the highest scores. But the decision on the basis of this method is likely to be arbitrary as this involves subjectivity. The selection will hardly be free of allegation of favouritism or corruption.1 Moreover, this method implicitly makes a dubious assumption that the government has better information about the firms’ prospects than the firms themselves.

So, beauty contest is not a better substitute of ascending auction and perhaps worse under the likelihood of corruption. Furthermore, the ascending auction is defended on the argument that it not only generates highest revenue for the government but the license fee being a sunk cost does not figure in the profit maximization condition for the firms and does not raise price. When deciding how to set prices, the firm rationally takes account of its own forward looking costs and revenues and the likely behaviour of other firms and does not raise price above the marginal cost notwithstanding a high license fees, since the other firms can undercut. Hence the quantum of the license fee cannot affect prices.

But this argument does not take into account the fallout of the high license fee on the cost of borrowing and on investment in turn. If the high license fee raises cost of borrowing, there can be a problem, given the investment requirements of the industry. In case of ascending auction, the licensing fee is likely to be high and it adversely affects the infrastructural investment, like erection of tower, laying of the optic fibre line, developing the network, etc. According to the technical experts of the telecom industry, investments in telecom networks can be divided into the following functional elements like terminal equipment, access network, switching, transmission and development of long line facilities. Unless these investments are made at an appropriate level, the services of the telecom industry cannot be efficient. In fact India has already been facing this problem with not only a vastly inadequate rural telecom infrastructure but also with a strained urban telecom infrastructure.

THE STUDY

In this paper, we consider a duopoly game between two sellers, who have been granted spectrum license by the Government. The firms play quantity as well as capacity game that again determines the quality of services. This is basically the Cournot game although a couple of differences that we are incorporating are as follows: (1) The licensing fee and the associated interest costs are introduced that has bearing on capacity choice; (2) The quality of service parameter is introduced in the analysis. This quality of service depends on the capacity installation. The model shows that capacity installation is smaller on account of the adverse impact of high license fee and the fallout is deterioration in quality of services. We shall also introduce the role of the social planner who has to design a suitable program to optimize the market. We suggest a policy of spectrum auction where a part of the license fee should be used for subsidization of the infrastructural investment.

LITERATURE REVIEW


The issue of pros and cons of spectrum auction have received wide attention in the literature. Zheng (2001)

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1 We can recall the corruption issue involved in 2G license distribution by discretionary measures in India.
shows how a high bid for spectrum auction can lead to bankrupt winners. But Cave and Valetti (2000) have shown that the spectrum auction through ascending auction need not necessarily be bad as the license fee is only a sunk cost. Haan and Toolsema (2003) show that consumers’ price may be lower under spectrum auction than under beauty contests. On the other hand, Burguet (2005) shows how the beauty contest method is a better approach to spectrum distribution than the auction of licensing. It is thus observed that the relative merits and demerits of spectrum auction and beauty contests have remained unresolved and this paper is an addition in this large body of literature. The one area that is perhaps not attended to in the literature and which the present paper addresses is the possibility of using high license fee for subsidizing the investment in the telecom sector.

THE MODEL

The government auctions the spectrum, which the duopoly firms buy on commitment of a payment of a licensing fee ($L_F$). We assume that this payment involves a fixed cost ($F$) on the firms. After acquiring the license, the telecom firms approach banks that work in a competitive environment for financing the related investments ($V$). The firms work in a market where there is a demand for telecom services. The business may be good or bad and the lending bank assigns subjective probability $p$ and $1 - p$ respectively for good and bad state. The bank charges an interest and a profit level that is driven down to zero (only normal profit that is included in cost) by competition determines this interest rate. Bank gets an interest ($i$) in case of good business but gets a realized value of collateral ($K$) in case of failure. This realized value is a fraction ($\alpha < 1$) times the loan amount $V$. We assume that this fraction on collateral is inversely related with the ratio ($L$) between licensing fee and firm’s total asset and higher the $L$, lower is the realizable value of collateral ($K$). This happens because the realized value of collateral falls when a major part of it is blocked in debts on account of high licensing fee.

We assume that the authority gives license to two firms that play Cournot game to serve a demand for telecom services. For simplicity, this demand is assumed to be linear.

The Game

- In the beginning, the firms participate in an auction game and get the license on payment of license fees to the government mainly with borrowed money.
- The firms, having secured the license, approach the banks and take loan for the investment.
- The game on production between two firms begins now. The firms determine the quantity and the quality of service, taking into account the license fees. The price is determined as per the quantity of services. We apply the backward induction method to determine the license fees. The firms calculate the possible profit in the second stage and bid for the license. The regulatory authority gives license to two firms that play the Cournot game in the second stage. The firms that bid but do not get the license are out of the market and we do not consider their case.

Licensing Acquisition

Under Cournot game with identical quality of services, we consider a linear demand function:

$$ P = a - bq $$

Given marginal cost $c$, the firm makes an expected profit

$$ \pi = \left( \frac{a - c}{3b} \right) \left( \frac{a + 2c}{3} \right) $$

Hence there will be a bidding game between firms for which the license fee will equal the expected profit. We shall now see how $c$ turns out to be in the licensing scenario.

After getting the license, the firm decides on the infrastructure and investment by a capacity game with the help of loan from the banks. Once the capacity is built up, services are provided. Here we assume that there is a quality of services (QoS) parameter $\theta \in (\bar{\theta}, \tilde{\theta})$, where $\bar{\theta}$ is the lowest (most inefficient) limit of QoS and $\tilde{\theta}$ is the highest limit. We therefore postulate a relation $V = \theta q$, where $V$ represents the infrastructural investment. The relation shows that given $V$, if quantity of services is more, quality must deteriorate.

Bank Lending to Telecom Firms

Once license is granted and the firms approach the banks for loan for infrastructural development, we get the following bank lending scenario:
\[ \pi_B = p_iV + (1 - p)\alpha V - rV \]  
\[ \alpha = s - hL > 0 \]  
\[ \pi_B = p_iV + (1 - p)(s - hL)V - rV \]  
\[ L, s, h = \in [0, 1], p > (1 - p)\alpha, p \geq 1/2. \]

where, \( V \) = Amount borrowed for infrastructural investment, \( r \) = interest on deposit, i.e., cost of bank fund, \( i \) = interest charged to the borrower, \( p \) = probability of good business and recovery of interest, \( \alpha \) = realizable fraction of the loan in case of loss, inversely linked to \( L \) = License fee – Firm’s asset ratio, given by a linear relation, as in (1). Thus, it is assumed that in case of successful business, the bank gets the interest while in case of bad state, the bank realizes the return from collateral\(^2\). Obviously,

\[ p_iV > (1 - p)(s - hL)V \]

Under competitive condition, the profit of the bank will be zero. So, we get the bank profit as

\[ \pi_B = p_iV + (1 - p)(s - hL)V - rV = 0 \]

Then,

\[ i = \frac{r - (1 - p)(s - hL)}{p} \]

(3)

The firm’s interest cost is therefore

\[ iV = \frac{rV - (1 - p)(s - hL)V}{p} \]

(4)

Equation (4) shows that firm’s interest cost rises with the rise in licensing cost. This is because the lending bank raises the interest rate with increased risk of default on account of higher license fee. Once loan is received, the quantity and quality of services game begins. We consider a Cournot quantity game here.

**Sub-Optimal Quality of Telecom Services**

In this section, we introduce quality of service (QoS) in our model. As ICRA Rating Finance (2009) observes, with one of the lowest tariff rate in the world, the scope for price competition is limited in the Indian telecom industry. Given this fact, QoS would become the prime distinguishing factor among the competing companies, although a rapidly increasing subscriber base would make it difficult for telecom operators to maintain the minimum level of QoS. But given the low switching cost on the part of the consumers with mobile number portability, operators will require additional infrastructure in their existing areas of operation to be able to offer better QoS in order to retain existing subscribers. In order to take this factor into account, we introduce a \( \theta \) that represents the QoS factor. If one firm has higher \( \theta \), it attracts more customers from the other firm. So, in equilibrium, \( \theta \) of two firms is equal, representing a homogenous good market. In this set up, we derive the following proposition:

**Proposition 1:** In the final game in the market, given the above interest cost pattern, there is a Cournot game solution but with sub-optimum quality of services. There exists an inverse relation between license fee and quality of services but the relation between license fee and quantity is ambiguous.

**Proof:** See Appendix I.

We give here the intuitive explanation of the above result. While the firms play the Cournot quantity game, they can also adjust capacity to offer a better quality of services. But since the firm has a huge liability of license fee, they are forced to face a high rate of interest. Therefore, they make less than optimum investment although they know that higher capacity leads to better service and better demand. Their profit maximization exercise results in a sub-optimal capacity and quality level.

The above result also shows that higher the license fee, lower is the quality of services. If the regulatory authority now insists on higher \( \theta \), the firm claims bankruptcy. This is *winner’s curse* in one sense. The implication of this sub-optimal capacity investment \( V \) is serious. If investment in physical infrastructure is not up to the mark, the customers will face more and more problems with regard to quality of services.\(^3\) Higher the license fee, more acute is the problem.

There are two circumstances where the licensing fee can be high. The first case is winner’s curse. The other possibility is the bankruptcy under limited liability. Zheng

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\(^2\) Equation (2) assumes that the banks are risk-neutral. The first two terms represent expected interest revenue of the bank and the last term represents actual cost of deposits. So, the profit here is the expected profit, which is assumed to fall to normal level by competition.

\(^3\) In India, both GSM operators and CDMA operators hurl charges against one another that the other makes underinvestment in capacity build up and hence misuses spectrum, granted by the authority.
(2001) shows that under limited liability, cost of bankruptcy is low and then the firm bids more and is likely to be a broke winner. But we show a third possibility where the opportunity of low capacity investment and maintaining low quality of services can lead to a high bid. If the regulator likes the firms to charge an optimum price, i.e., adopt marginal cost pricing, then the firms will increase \( q_i \) by overuse of the fixed capacity and the quality of services will decline.

Our second result that the effect of increased license fee on output is ambiguous is consistent with the result of the literature that the price need not necessarily rise with a higher auction bid. Haan and Tulsema (2003) get this result but their result is based on the argument that debt financed auction leads to a more fierce competition among the firms. But here compromise on quality is the route to fixity of price.

In the next part, we assume that the social planner is aware of this fall out of license fee on investment and quality of services and therefore tries to pursue a policy that takes care of the social objective of maximization of growth.

**Planner’s Objective**

Suppose the social planner has the objective of maximizing the growth rate, then we get the following result:

**Proposition 2:** It may not be optimal to maximize the license fee. The optimal policy may be \( 0 < L_F^* < L_F^{\text{max}} \) where \( L_F \) is the absolute value of license fee.

**Proof:** We assume a simple framework where the growth rate \( (w) \) is positively related to the average infrastructural investment in telecommunication \( v = \frac{V}{q} = \theta \) where \( \frac{dv}{dL_F} < 0 \), and a government investment \( (I) \) for development, financed by part of the license fee. So, we have the following scenario:

\[
w = w(I, v) \quad w_I > 0, w_v > 0.\]

Let \( I = I(L_F) \)

The first order condition of maximization of growth rate by using \( L_F \) is

\[
\frac{dw}{dL_F} = \frac{\partial w}{\partial I} \frac{dI}{dL_F} + \frac{\partial w}{\partial v} \frac{dv}{dL_F} = 0
\]

The first term is positive and the second term is negative. So, their sum can be zero at an interior solution, implying less than maximum license as the optimum policy of the government.

The social planner has to find out the optimum license fee that also optimizes the infrastructural investment in the telecommunication sector.

We can venture a diagrammatic representation of the above optimizing exercise (Figure 1).

From the growth maximizing condition we get

\[
- \frac{\partial w}{\partial v} \bigg| \frac{\partial w}{\partial I} \bigg| \frac{dI}{dL_F} \bigg| \frac{dv}{dL_F} < 0
\]

The LHS term is the slope of the iso-growth curve, which we assume to be convex from below, under the standard assumption of diminishing MRTS. The right hand side is the slope of the license fee-led private investment – public expenditure trade-off curve, which is shown to be downward sloping. If now rate of increase in \( I \) falls as \( L_F \) goes on rising and rate of fall in \( v \) rises as \( L_F \) goes on rising, we get that the slope of the private investment – public expenditure trade-off curve approaches 0 as \( v \) tends to zero. On the opposite side, we assume that the slope

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4 In India, the price of telephone call is falling but a report of TRAI states that the point of intersection congestion levels is more than 40% in 72 out of 193 locations. In many cases, there is disconnection in between the talk. In many other cases, the call recipient does not receive the caller’s voice in the first connection and has to make the second call and pay for both the calls. For some mobile services, voice audibility is a serious problem. In a study of US telecom services by Power & Associates, it is observed that broadband and high-end data customers are experiencing decline of services.
of the private investment–public expenditure trade off curve approaches $\infty$ as $v$ tends to $v_{\text{max}}$. Since $0 < \frac{\partial w}{\partial v} \frac{\partial w}{\partial I} < \infty$, with the assumption of continuity, we get a tangency solution, implying $L_F^* < L_{F_{\text{max}}}^*$.

The above policy suggests an optimum license fee that is less than the maximum. But under spectrum auction, we get maximum license fee. So, we prescribe an alternative policy of auction and subsidization that can avoid the problem of low investments while retaining the advantages of auction.

**Auction cum Subsidy Programme: An Alternative Policy**

We can now think of another strategy option under which the government charges a higher license fee but uses a part of it for helping the telecommunication sector. Let us capture this by introducing a subsidy to the telecommunication industry, financed with the help of license fee. Implication of this subsidy is that the government precludes the use of a part of the license fee for its own use.

The firm gets a unit subsidy at the rate $R$, that is positively related to $\theta$, i.e. higher the quality of services, more is the subsidy. So, we have $R = R(\theta)$ and $R'(\theta) > 0$.

We consider below the results in this subsidy-included Cournot game:

**Proposition 3:** Under certain conditions, a combination of license fee and subsidy leads to an increase in average investment with increase in license fee, and therefore it is optimal to make an optimum combination of license fee and subsidy to maximize telecom investment and economy’s growth rate.

**Proof:** See Appendix II.

While the result is proved mathematically, we can attempt an intuitive explanation of the above result. If the government decides to subsidize the infrastructural investment in telecommunication, that will be taken into account in the profit maximization behaviour of the firms. This will ensure the capacity addition and improvement of quality of services. It is shown in our mathematical result that subsidy amount will be incorporated into the auction bid and profit will be driven down to the minimum level.

The payment of subsidy out of license fee accretion reduces the government’s ability to spend for development in non-telecom sector but given the trade-off nature of such expenditure and investment in telecom in contribution to development, it is optimal to spend a part of the license amount for subsidization of the telecom investment. The ability of the government to manipulate the subsidy rate does the trick of promoting maximum growth.

The subsidization for telecom infrastructure is not a novel idea. It is practised both in developed and developing economies. USA has a Universal Services Fund (USF) system under which companies operating in high service cost area are subsidized (Chiang, Hague and Jamison, 2007). But of late, the US Federal Communications Commission has been making a plan to orient the subsidy towards broadband deployment. One World Bank Report (2001) shows that in Peru, subsidy is paid to install equipments in targeted rural areas subject to compliance of performance standard. The study shows that this helps to mobilize private investment double the amount of subsidy. In India, there has been a demand that rural telecom towers be more solar-powered than diesel-powered but that needs huge investment.

Here comes the case for utilization of a part of the license fee for subsidization of such infrastructural development in telecom industry.

**CONCLUSION**

We, in a duopoly set-up, derive the conclusion that high license fee leads to a fall in the average physical investment in the telecommunication industry on account of the reaction of the bank to take into account the high fees in determining the rate of interest, chargeable to the telecom operators. As a result, the profit maximizing behaviour of the firms leads to lower infrastructural investment per unit of services and compromise on the quality of services. The neglect of physical investment tells upon the health of not only the telecommunication industry but also of the economy as a whole. In this case, the social planner may decide to reduce license fee but that will reduce the budgetary receipt of the government and at the same time fail to provide the regulator with any instrument to promote investment. A better strategy, therefore, may be to earmark a
part of the license fee for government support towards the expansion of the telecommunication industry. This support can help the government to monitor actual performance, which a blanket reduction in license fee does not. Although these results are model specific, the assumptions on which they are based are fairly realistic and therefore, they have relevance in policy formulation.

We should of course sound a note of caution. Subsidies have a tendency of misutilization. Hence it should be strictly performance-based and should have a link with the spectrum auction fee paid. It should not be a blanket subsidy but a task-oriented one. The regulatory authority has an onerous task in this regard to ensure that the subsidy is put to desired use.

Finally, the limitation of the study should not be overlooked. It is a model-based theoretical study, dependent on a slew of simplifying assumptions. The issue may need a broad-based empirical study in order to take into account the ground level reality. That can be a future research agenda.

**Appendix I: Proof of Proposition 1**

a) Given the market demand function

\[ p = a - bq \]  \hspace{1cm} (A-1)

We introduce the quality aspect of services by considering the following demand function for firm 1 and firm 2 as under:

The demand faced by firm 1 is

\[ P = a - bq + g(\theta_1, \theta_2) \text{ where } q = q_1 + q_2 \]

\( \theta_1 \) is the quality of services level of firm 1.

\( \theta_1, \theta_2 \) are the lower and upper limit of \( \theta_1 \) and if given \( \theta_2, \theta_1 \) rises within these two limits, demand for firm 1’s service rises.

The demand faced by firm 2 is

\[ P = a - bq - g(\theta_1, \theta_2) \]

\( g = g(\theta_1, \theta_2) \) has the following specifications:

\( g_1 > 0 \) for \( \theta_1 \in [\theta_2, \theta_1] \) and \( g_1 = 0 \) for \( \theta_1 = \theta_2 \)

\( g_2 < 0, g_{11} < 0, g_{12} \geq 0, g_{22} > 0 \)

We further assume \( g = 0 \) for \( \theta_1 = \theta_2 \)

This implies that demand for firm i’s service rises at diminishing rate as service quality of firm i increases given firm j’s quality but there is a limit to the increase in quality.

In our following study of Cournot game, we take the interest cost of the earlier analysis and get the following profit functions of the two firms:

\[ \pi_1 = aq_1 - bq_1^2 - bq_1q_2 + g(\theta_1, \theta_2)q_1 - \frac{r\theta_1q_1 - (1-p)(s-hL)\theta_1q_1}{p} - F \]  \hspace{1cm} (A2)

Let \( \frac{r - (1-p)s}{p} = \beta \) and \( \frac{h(1-p)}{p} = n \)

\[ \pi_1 = aq_1 - bq_1^2 - bq_1q_2 + g(\theta_1, \theta_2)q_1 - (\beta + nL)\theta_1q_1 - F \]

\[ \pi_2 = aq_2 - bq_2^2 - bq_1q_2 - g(\theta_1, \theta_2)q_2 - (\beta + nL)\theta_2q_2 - F \]
The first order conditions are:

\[
\begin{align*}
\frac{\partial \pi_1}{\partial q_1} &= a - 2bq_1 - bq_2 + g - (\beta + nL)\theta_1 = 0 \quad \text{(A3)} \\
\frac{\partial \pi_2}{\partial q_2} &= a - bq_1 - 2bq_2 - g - (\beta + nL)\theta_2 = 0 \quad \text{(A4)} \\
\frac{\partial \pi_1}{\partial \theta_1} &= g_1q_1 - (\beta + nL)q_1 = 0 \quad \text{(A5)} \\
\frac{\partial \pi_2}{\partial \theta_2} &= -g_2q_2 - (\beta + nL)q_2 = 0 \quad \text{(A6)}
\end{align*}
\]

From the above equations, we get optimum levels of \(q_1, q_2, \theta_1\) and \(\theta_2\). From (A5), we get \(g_1 > 0\) and thus an interior solution for \(\theta_1\) and \(\theta_2\) exists. This implies that \(\theta^*\) will be settled at sub-optimum level that will lead to sub-optimal quality of telecom services.

We can next consider the effect of a change in licensing fee on the quality of service and quantity. Since we get a symmetric equilibrium with \(q_1 = q_2\) and \(\theta_1 = \theta_2\), we can consider equation (A3) and (A5) for two variables \(\theta\) and \(q\).

Total differential of (A5) gives

\[
\frac{d\theta^*}{dL} = \frac{n}{g_{11}} \quad \text{< 0} \quad \text{(A7)}
\]

under the condition \(g_{11} < 0\).

Again from (A3),

\[
\theta^* = \frac{1}{3b} \left\{ a - (\beta + nL) \theta \right\}
\]

\[
\frac{dq^*}{dL} = \frac{1}{3b} \left\{ -n\theta - (\beta + nL) \frac{d\theta}{dL} \right\}
\]

Given that \(\frac{d\theta}{dL} < 0\), the bracketed term has no definite sign.

This proves Proposition 1.

**Illustrative Example**

Assume \(g = \frac{\theta_1^{1/2}}{\theta_2^{1/2}} - 1\)

Then, solving the first order conditions of profit maximization, we get

\[
\theta_1^* = \theta_2^* = \frac{1}{2(\beta + nL)} \quad \text{(A8)}
\]
\[ q_1^* = q_2^* = \frac{a - 1/2}{3b} \]  

(A9)

It is clear that the relation between \( \theta \) and \( L \) is inverse but there is no relation between \( q \) and \( L \) in this example.

**Appendix II: Proof of Proposition 3**

The demand function is given by

\[ \pi_1 = a q_1 - b q_1^2 - b q_1 q_2 + g(\theta_1, \theta_2) q_1 - (\beta + nL)(1 - R(\theta)) \theta q_1 - F \]  

(A10)

The first order condition for firm 1 is

\[ \frac{\partial \pi_1}{\partial q_1} = a - 2b q_1 - b q_2 + g(\beta + nL)(1 - R(\theta)) \theta q_1 = 0 \]  

(A11)

Similarly,

\[ \frac{\partial \pi_1}{\partial q_2} = a - b q_1 - 2b q_2 + g(\beta + nL)(1 - R(\theta)) \theta q_2 = 0 \]  

(A12)

\[ \frac{\partial \pi_1}{\partial \theta_1} = g q_1 - (\beta + nL)(1 - R(\theta)) \theta q_1 + R_1'(\beta + nL) \theta q_1 = 0 \]  

(A13)

\[ \frac{\partial \pi_2}{\partial \theta_1} = g q_2 - (\beta + nL)(1 - R(\theta)) \theta q_2 + R_2'(\beta + nL) \theta q_2 = 0 \]  

(A14)

From which we can solve for \( q^*_i(R_i) \) and \( \theta^*_i(R_i), i = 1, 2 \).

This shows the optimum combination of quality of service and subsidy.

**Illustrative Example**

Assume \( g = \frac{\theta_1^{1/2}}{\theta_2^{1/2}} - 1 \)

Solving for the optimum value from the first order conditions, we get

\[ \theta_1^* = \frac{1}{2 (\beta + nL)(1 - R(\theta)) + R_1'} \]  

(A15)

This shows that the social planner can optimize quality of services by proper use of license fee and subsidy.

Writing the profit equation in terms of \( V_i \) from \( \frac{V_i}{\theta} = q_i \) and taking \( a = b = 1 \)

and \( R(\theta) = m(\theta) q \), we get the following profit function

\[ \pi_i = \left( 1 - \frac{V_1}{\theta} - \frac{V_2}{\theta} \right) \frac{V_1}{\theta} - (\beta + nL) V_1 + m(\theta) \frac{V_1}{\theta} \]
Here \( g = 0 \) since \( \theta \) is optimized.

Solving for \( V_1 \) after a Cournot game, we get

\[
V_1 = \frac{\theta}{3} - \frac{\theta^2 (\beta + nL - m/\theta)}{3}
\]

(A16)

With above \( V_\rho \), we get

\[
\pi = \frac{1}{9} \left( 1 + m - nL\theta - \beta\theta \right)^2
\]

(A17)

[subscripts ignored for notational simplicity]

The social planner calculates and enforces \( m \) so that \( \pi \) is maximized at the socially optimum level of \( \theta \). This gives Nash equilibrium with \( \theta = \theta^*, \: m = m^* \).

For this purpose, we consider the following first order conditions:

\[
\frac{\partial \pi}{\partial \theta} = -\frac{2}{9} (1 + m - nL\theta - c\theta) (\beta + nL_F + m')
\]

(A18)

\[
\frac{\partial \pi}{\partial L_F} = \frac{2}{9} (1 + m - nL_F\theta - \beta\theta)(- n\theta) < 0
\]

(A19)

Given that \( \frac{\partial \pi}{\partial L_F} < 0 \), \( L_F \) can only minimize \( \pi \). So, ignoring the possibility of winner’s curse, we assume that the firm pays only that \( L_F \), say \( L_F \), that gives minimum profit level, say \( \pi = 0 \).

Then, from (A17), \( \theta^* = \frac{1 + m}{\beta + nL_F} \)

(A20)

The social planner can manipulate \( m \) to keep \( \theta \) at optimum level, given \( L_F \).

This shows that \( \theta \) and \( v \) can be increased by means of subsidy at an unchanged \( L_F \). Subsidy of \( m \) increases infrastructural investment and as a result the quality of services also improves.

REFERENCES


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