Financial decision-making is not straightforward, in part, because such decisions generally involve comparing financial assets the payoffs from which are subject to risk and uncertainty.

Given that situation, two questions naturally arise: How do economic agents go about the business of making choices in the face of risk and uncertainty? And, how should economic agents make choices in the face of risk and uncertainty?

This paper concentrates on the first of these questions and discusses some of the main attempts made by economic theory to understand how economic agents go about the business decision-making under conditions of risk and uncertainty.

Theoretical possibilities considered in the context of decisions under conditions of risk include: Expected value maximization, Expected utility maximization, Rank dependent utility maximization, Prospect theory, and the Topology of fear approach to decision-making in the face of catastrophic risk. This paper also considers empirical tests of these theoretical possibilities and some of the anomalies and responses thrown up by those tests such as: Allais Paradox, Discovered Preference Hypothesis, and the choice behaviour of CEOs when faced with risk.

The paper concludes with a brief excursion into choice under uncertainty where, unlike in risky choice situation, the existence of objective probabilities over states of the world cannot be relied on. In that context, the author briefly canvases the Subjective Expected Utility approach – which is unable in general to account for ambiguity aversion – Choquet utility, Wald’s Multiple Priors, and the Case Based approach.

This paper highlights the fact that the rich and fascinating field of decision-making under risk and uncertainty is characterized by a constant interplay between theoretical conjecture, empirical testing, and theoretical refinement. Such interplay is mirrored by this paper and contributions in the Colloquium Section of this Issue, where the thoughts of practitioners and academics interact.
“One of the most intriguing areas of research in finance is that which examines and seeks to explain how and why individuals make decisions. This research is aimed at predicting accurately decisions that will be made given certain conditions and circumstances. The most commonly accepted model of rational choice today is the theory of utility of wealth developed by von Neumann and Morgenstern (1953). Questions have arisen recently, however, concerning the completeness of this theory.”


“There can be few areas in economics that could claim to have sustained such a rich interaction between theory and evidence in an ongoing effort to develop theories in closer conformity with the facts. Considered together, the accumulated theory and evidence present an opportunity to reflect on what has been achieved.”

Chris Starmer (2000, p. 332)

The valuation of financial assets is a primary focus of Finance Theory because it is asset values that guide asset allocation, (see the discussion in Wang, 2008). The valuation of assets is not a straightforward exercise, however, because asset values typically derive from future payoffs. Such payoffs are subject to risk and are uncertain in both timing and extent. Risk, uncertainty, the attitudes of economic agents to them, and the way that agents make their demand and supply decisions under conditions of risk and uncertainty are therefore crucial in determining asset values.

Consequently, this paper discusses some of the attempts made by economic theory to understand choice and decision-making under conditions of risk and uncertainty. This is a rich and fascinating field of endeavour where the constant interplay between empirical testing and theoretical refinement is clearly visible – an interplay interestingly mirrored in the Colloquium Section of this Issue, where contributions by practitioners and academics interact.

SETTING THE SCENE

The paper begins with a comparison of choice under certainty and choice under uncertainty as a way of coming to conceptual grips with the choice under uncertainty situation of a decision-maker.

Choices under Certainty vs Uncertainty

The standard model of choice under certainty involves the idea of: (i) a choice set, C, to which the decision-maker has direct access; (ii) an ordering \( \preceq \), defined over the choice set that captures a decision-maker’s preferences; and (iii) a behavioural hypothesis that if the decision-maker chooses option \( c^{*} \) out of C, then there is no other option in C ranked more highly by the decision-maker than is \( c^{*} \) – although there may be ties.

Choice under uncertainty enriches the choice under certainty picture by supposing that decision-makers now have only indirect access to the set, C, now referred to as the set of consequences. Decision-makers have direct access only to a set A of actions. It is in the set of actions A that agents have to operate, and over which they somehow have to define a preference ordering, \( \preceq_{A} \).

Connecting A and C via the World

The set A is connected to the set, C, by the world, W. The idea is that if a decision-maker takes an action \( a \in A \), then, depending on the state or mood of the world, that action may lead to any one of a number of consequences. This is expressed by saying that \( W(a) \subseteq C \), meaning that a given action may be associated with a subset, or even the whole set, C.

This formulation illustrates a central aspect of decision-making under uncertainty, namely, a decision-maker does not know in advance, the consequences of a given action. Contingent on the state of the world, a given action may be mapped to any one of a number of consequences – see Figure 1 for an illustration.

Figure 1: The World Maps Actions into Consequences
In this way of looking at things, uncertainty is thought of as meaning: (i) that decision-makers have only indirect access to the space they are really interested in, the space of consequences, $C$; (ii) their access is through the space of actions $A$; (iii) the mapping $W: A \rightarrow C$ is multivalued, i.e. it is a correspondence and not a function. If $W$ is a function then for each $a \in A$, $\exists! c \in C$ such that $W(a) = c$, and we have choice under certainty.

Remark: The elements in the spaces of actions, $A$ and consequences, $C$ can be very general. The case where the consequences and/or actions are monetary is dealing with financial decisions.

The Decision-maker’s Knowledge: Objective Probabilities

An interesting aspect of choice under uncertainty concerns the decision-maker’s knowledge of the world in which they have to operate. In this, there are two leading cases. Firstly, it may be that the decision-maker has access to objective probabilities about the likelihood of a particular state of the world, $W_1, \ldots, W_n$ occurring. This then provides connections between a given action and elements in the set of consequences.

Such a situation is common in, for example, casino games. It can be expressed as the objective probability, $p(c | a)$, of a particular consequence $c \in C$ being experienced when a particular action $a \in A$ is taken. By letting $c$ range over all of $C$ and $a$ range over all of $A$, the decision-maker can get a probabilistic picture of the uncertainty they face and hence information about the map $W: A \rightarrow C$.

Situations in which a decision-maker has objective probabilistic information about the way actions are connected to consequences will be referred to as Risk. In such circumstances, it can also be useful to think of actions as probability distributions over consequences. It may be written as $a_k = (c_{1k}, c_{2k}, \ldots, c_{mk})$ where the notation $p_{1k}$ indicates the probability that consequence $c_i$ will be experienced if action $a_k$ is chosen for $i = 1, \ldots, n$ and $k = 1, \ldots, m$.

Example 1: Suppose an individual is given the option to roll a fair die, $(a_r)$, or to walk away $(a_w)$, the space of actions $A$ is then $\{a_r, a_w\}$. If they decide to roll the die, they are promised a loss of $1\text{m}$ if 1 comes up; a gain of $2\text{m}$ if 2 comes up; a loss of $3\text{m}$ if 3 comes up; a gain of $4\text{m}$ if 4 comes up; a loss of $5\text{m}$ if 5 comes up and a gain of $6\text{m}$ if 6 comes up. If they walk, they gain nothing and lose nothing;

$$
p(-1 | a_r) = p(+2 | a_r) = p(-3 | a_r) = p(+4 | a_r) = p(-5 | a_r) = p(+6 | a_r) = \frac{1}{6}
$$

The decision-maker can then make a probabilistic analysis of the world they are in, as in equation which gives them a picture of the map $W: A \rightarrow C$, as in Figure 2.

![Figure 2: The Map W: A→C of Example 1](image)

The Decision-maker’s Knowledge: No Objective Probabilities

A second leading case involves situations where the decision-maker does not have complete - or any - objective information about the nature of the world they are in. They have no idea, even in probability terms, how actions and consequences are connected, i.e. they are uncertain about the nature of the map, $W: A \rightarrow C$. Such a decision context is referred to as Uncertainty.

As is traditional, the author begins with trying to understand the selection of actions by a decision-maker in the special case of Risk. Equipped with some important insights gained by studying that case, he moves to consider decisions under Uncertainty. Collecting the thoughts so far gives the following:

**Definition 1 (Decision under risk and uncertainty):** Decisions under risk or uncertainty involve making choices between actions that yield consequences contingent on realizations of a priori unknown states of the world. In the case of decisions under Risk, agents have complete knowledge of the objective likelihood of each state. Under conditions of uncertainty, that is not the case.
The author follows the tradition in the literature of considering this problem firstly in the context of decisions under risk and then taking what we learn there to the general case of decisions under uncertainty.

**The Basic Research Question**

Given this setting, the central, fascinating, theoretical problem in the choice under risk and uncertainty literature is: *How do agents order the set of actions available to them?* In thinking about this question, it is important to be aware of the mindset that a decision-maker is imagined to have, namely that actions are not inherently interesting; they are viewed merely as means to consequential ends. It is the consequences that are inherently interesting.

Thus the mindset brought to the task of theorizing about how people go about ordering a space of risky actions is to suppose that the actions themselves are not inherently interesting. So, for example, if a property developer has to appear every night for the next month on a very tedious television programme in order to drum up business for their latest real estate development, they will take that action – even though they would much prefer to take the action of relaxing in a spa bath with a bottle of Champagne.

This contrasts with a person who just loves being on TV and will take any opportunity to appear on any programme for free, and no matter what the consequence. The latter person has his focus on the space of actions, with no regard for the space of consequences. The former person is intently focused on the space of consequences and is trying to select from the space of actions in such a way as to get themselves to a good consequence.

**TWO FUNDAMENTAL BEHAVIOURAL HYPOTHESES: EVMH AND EUMH**

Given an agent’s ordering of the space of consequences, $\preceq_c$, along with their probabilistic understanding of the world, $W$, how do they go about making a preference order, $\preceq_A$, over the space of risky actions available to them?

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2 This may be thought of as the ‘consequentialist point of view. See the discussion of this approach in Hammond (1988).

3 We refer to the elements of the set as *risky actions*. Other terms found in the literature include ‘lotteries’, ‘gambles’ and sometimes, ‘prospects’.

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**The Expected Value Maximization Hypothesis (EVMH)**

Let a decision-maker be in a risk situation in which $i = 1, ... , n$ possible consequences $c_1, ..., c_n$ make up the space of consequences $C$. Let the space of actions $A$ consist of $k = 1, ..., m$ elements $a_1, ..., a_k$. One possibility is that decision-makers order the space of actions by calculating the expected value of each action.

**Definition 2** (Expected value of an action): In the case where space of consequences is a non-empty finite set, the expected value of action $a_k \in A$ is $EV(a_k) = p_{1|k}c_1 + ... + p_{n|k}c_n$ with $p_{i|k}$ being the probability of consequence $i$ being enjoyed, when action is taken.

The Expected Value Maximization Hypothesis: In a risk situation, if $a_* \in A$ is chosen, then $EV(a_*) \geq EV(a)$, $\forall a \in A$, so that a decision-maker will choose an action $a_*$ that is not dominated, in expected value terms, by any other action in $A$.

**Is the EVMH Theoretically Viable?**

The answer is ‘yes’ if: (i) there are conditions under which expected values exist; and (ii) there are conditions under which maximum expected value exists. There are such conditions but they are not perfectly general.

As far as (i) is concerned, the existence of an expected value is equivalent, in the case of a discrete consequence space to the absolute convergence of the corresponding series. A series converges absolutely if the sum of the series remains finite when the terms in the series are replaced by their absolute values. Well-known cases where expected values can’t be defined, include the case where an action is defined by a Cauchy distribution over consequences, e.g. $p(c | a) = 1/\pi (1 + c^2)$.

As for (ii), if the set of actions is assumed to be non-empty and compact, then the expected value function when it exists being affine and continuous will take a maximum value.

**Is the EVMH Empirically Viable?**

There are well-known empirical objections to this hypothesis, and they will be discussed momentarily. Before
As Machina (2008) notes, EUMH is, one of the most influential ideas in decision theory:

“The expected utility hypothesis is the predominant descriptive and prescriptive theory of individual choice under conditions of risk or uncertainty … The approach can be applied to a tremendous variety of situations, and most theoretical research in the economics of uncertainty, as well as virtually all applied work in the field (for example, insurance or investment decisions) is undertaken in the expected utility framework.” (Machina, 2008; p. 130).

Similarly, Schoemaker (1982) and Camerer (1989) argue that expected utility theory is the foundation of the economics of uncertainty and the focus of much research and application in decision theory, finance, and psychology. Apart from directly addressing the question of how agents order the set $A$, another attractive aspect of the expected utility formulation is that it allows us to capture a decision-makers attitude to risk, through the concavity (or convexity) of their Bernoulli utility function $u_B$.

Given the centrality of EUMH and the massive stimulus to the literature it has provided, this paper discusses EUMH along with some of its tests and alternatives.

### Is EUMH Theoretically Viable?

EUMH is theoretically viable if: (i) it can be shown that there are circumstances under which an expected utility function exists which represents an agent’s preference ordering $\preceq_A$ on the space of risky actions $A$; and (ii) the function takes a maximum in the space of actions.

As far as (i) is concerned, Proposition 1 will show that under the conditions about to be listed, an expected utility representation of $\preceq_A$ on $A$ is possible.

#### Axioms 1-5

Axiom 1: $\preceq_A$ is a binary relation on $A$

Axiom 2: $\preceq_A$ is complete on $A$

Axiom 3: $\preceq_A$ is transitive on $A$

Axiom 4: $\preceq_A$ is continuous on $A$ so for any $a, a', a'' \in A$ the sets $[0 \leq t \leq 1]a', [0 \leq t \leq 1]a''$ and $[0 \leq t \leq 1][ta + (1 - t)a'] \sim A [ta' + (1 - t)a'']$ are closed

Axiom 5: $\preceq_A$ satisfies independence on $A$ so that $\forall a, a', a'' \in A$, and $0 < t < 1$ we have $a \preceq_A a'$ if and only if $[ta' + (1 - t)a''] \sim A [ta' + (1 - t)a'']$.

#### Notes

5 See also the results in Cokely and Kelley (2009).

6 Axioms 1-5 are often referred to as ‘the von Neumann-Morgenstern axioms’.
Remark: Each of these conditions on preferences over risky actions has been subject to scrutiny. Of particular interest from our point of view are Axioms 4 and 5. Consider first the requirement that \( \preceq_A \) is continuous on \( A \). As Mas-Colell, Whinston, and Green (1995) note, continuity of \( \preceq_A \) on \( A \) means that sufficiently small changes in probabilities do not change the ordering of actions.

Example (Mas-Colell et al. 1995; p. 171): Suppose in \( C \), there are three possible consequences: \( c_1 \) = death by car accident, \( c_2 \) = staring at the wall at home, \( c_3 \) = a beautiful and uneventful car trip. Suppose the agent’s utility ordering on \( C \) be such that \( u_0(c_1) < u_0(c_2) < u_0(c_3) \).

Let \( A = \{ a_0, a_1 \} \) where \( a_0 \equiv \text{stay at home} \), \( a_1 \equiv \text{go driving} \). Writing out the actions gives us that: \( a_0 = (c_1, c_2, c_3, p_{1,0} = 0, p_{3,1} = 0), a_1 = (c_1, c_2, c_3, p_{1,1} = t, p_{2,1} = 0, p_{3,1} = 1 - t) \). Continuity of \( \preceq_A \) on \( A \) requires that if \( a_0 \preceq_A a_1 \) when \( t = 0 \) it will also be the case that \( a_0 \preceq_A a_1 \) for sufficiently small positive values of \( t \).

This clearly rules out possible preference orders where ‘safety first’ is prioritized and any non-zero chance of death would reverse the ranking of \( a_0 \) and \( a_1 \). Another way to put this is that continuity makes the preference ordering insensitive to small variations in the probability of catastrophic consequences. We return to this condition below.

As far as the independence axiom is concerned, the following observation by Oliver (2003) is pertinent:

“Independence implies that the intrinsic value that an individual places on any particular outcome in a gamble will not be influenced by the other possible outcomes (either within that gamble or within other gambles to which the gamble is being compared), or by the size of the probability of the outcome occurring. The axiom requires that, when comparing gambles, all common outcomes that have the same probability of occurring will be viewed by the individual as irrelevant.” (Oliver, 2003; p. 36).

Proposition 1: If a preference relation \( \preceq_A \) on \( A \) satisfies the conditions in axioms (1)–(5) and the space of consequences \( C \) is a non-empty finite set, then \( \preceq_A \) admits representation by a function that has an expected utility form, then \( a_k \preceq_A a_\ell \Leftrightarrow EU(a_k) = p_{1,k}u_0(c_1) + \ldots + p_{n,k}u_0(c_n) \leq EU(a_\ell) = p_{1,\ell}u_0(c_1) + \ldots + p_{n,\ell}u_0(c_n) \).


As for (ii), since expected utility functions are affine and hence continuous, if \( A \) is non-empty and compact, then there will be an expected utility maximum in \( A \).

This is given formal expression as follows:

Proposition 2: If a preference relation \( \preceq_A \) on \( A \) admits an expected utility representation where \( A \) is non-empty and compact, then there is an \( a \in A \) such that \( EU(a) \geq EU(\alpha) \), \( \forall \alpha \in A \).

Proof: As an expected utility function is linear, it is continuous. By a theorem of Weierstrass’ such a function has a maximum on non-empty, compact sets.

Hence, there is a non-empty set of conditions under which EUMH is theoretically viable.

Is the EUMH Empirically Viable?

As noted above, the theoretical foundation of the EUMH rests on the existence of a function that makes complete, transitive, mixture-continuous and independent preferences over risky actions representable by the expected utilities of those actions.

There is a vast amount of empirical work aimed at scrutinizing the EUMH – and the individual axioms on which it relies. A brief list of such work includes stretching from Mosteller and Nogee (1951), Allais (1953), Ellsberg (1961), Kahneman and Tversky (1979), Schoemaker (1982), Harrison (1994), Hey and Orme (1994), Camerer (1995) to Blondel (2002), Fan (2002), Oliver (2003), Schmidt and Neugebauer (2007), Harrison and Rutström (2009), Harrison, Humphrey and Verschoor (2010), Roger (2011), Birnbaum and Bahara (2012), Blavatsky (2012a), Huck and Müller (2012) and Cavagnaro et al. (2013). In the space available, only a very selective treatment of the literature is possible, focusing on some classic studies, but mostly more recent work and especially studies testing EUMH in a financial markets. As a prelude to doing that however, it might be useful to note the general negative implications of empirical findings for the empirical viability of EUMH up to about the year 2000:

“There is substantial evidence that EUMH is likely to be descriptively misleading in at least some important contexts and, given the accumulating evidence supporting, in particular, probability weighting and loss aversion, we have at least some well-grounded hypotheses about...
important factors generating departures from the standard theory” (Starmer, 2000; p. 376).

In principle, the failure of any of the conditions (1)–(5) given above could be responsible for an empirical failure of EUMH. In reality, it seems that one of the most potent sources of failure of EUMH seems to be inapplicability of the ‘independence axiom’, condition (5) which says: \( \forall a, a' \in A, a \leq_a a' \text{ if and only if } [u(a + (1 - t)a') - u(a')] \leq_t [u'(a' + (1 - t)a'')] \text{ for any } a'' \in A \text{ and all } 0 < t \leq 1 \). The earliest and one of the most potent demonstrations of the failure of independence is due to an experimental design known as the ‘Allais paradox’.

**The Allais Paradox**

As a canonical example of behaviour inconsistent with EUMH, and an example that has stimulated an enormous amount of subsequent work, consider the example of Allias (1953):

**The Allais Paradox**: Let \( \{c_1, c_2, c_3\} \) with \( c_1 = €500\text{m}, c_2 = €100 \text{m} \) and \( c_3 = €0 \). Agents’ are given two sets of actions \( A_1 = \{a_1, a'_1\} \) and \( A_2 = \{a_2, a'_2\} \) which they are asked to order, where:

- \( a_1 = (c_1, c_2, c_3); p_{1|1} = 0, p_{2|1} = 1, p_{3|1} = 0 \);
- \( a'_1 = (c_1, c_2, c_3); p_{1|1} = 0.10, p_{2|1} = 0.89, p_{3|1} = 0.01 \)
- \( a_2 = (c_1, c_2, c_3); p_{1|2} = 0, p_{2|2} = 0.11, p_{3|2} = 0.89 \)
- \( a'_2 = (c_1, c_2, c_3); p_{1|2} = 0.10, p_{2|2} = 0, p_{3|2} = 0.09 \)

Allias (1953) reported that in situations like this a majority percentage of respondents revealed the following preference orderings: \( a'_1 \preceq a_1 \) and \( a_2 \preceq a'_2 \). Such behaviour is inconsistent with the supposition that \( A \equiv \{a_1, a'_1, a_2, a'_2\} \) is being ordered using an expected utility function.

To see why, suppose that the ordering \( a'_1 \preceq a_1 \) and \( a_2 \preceq a'_2 \) is consistent with expected utility theory – and derive a contradiction as follows. Let \( u_B(500), u_B(100) \) and \( u_B(0) \) be the Bernoulli utility of the three consequences. The preference ordering \( a'_1 \preceq a_1 \) means, supposing an EU representation of \( u_B \) that:

\[
0.10 \times u_B(500) + 0.9 \times u_B(100) + 0.01 \times u_B(0) < 1 \times u_B(100)
\]

(2)

But if (3) holds, then \( a'_2 \triangleleft a_2 \). This contradicts the ‘empirical fact’ from the Allais experiment which revealed that \( a_2 \triangleleft a'_2 \). The author is therefore forced to drop the idea that an expected utility function represents \( u_B \) and that describes decision-making under risk.

Given the implications of the Allais paradox for such an appealing notion as EUMH it is not surprising that the paradox has been subjected to scrutiny along the following lines.

**Real vs Hypothetical Payoffs**

Harrison (1994; p. 223) argues that ‘the experimental evidence against expected utility theory is, on balance, either uninformative or unconvincing’ and that ‘pronouncements of the passing of expected utility theory are premature’. Burke et al. (1996) suggest that the reason for Harrison’s (1994) position is that his review of numerous influential experiments that find against EUMH lead him to conclude that many of the experiments are invalid because they fail to meet the requirements of salience or dominance suggested by Smith (1982)

8. Salience means that rewards in an experiment are systematically related to actual choices. Dominance means that rewards outweigh other motivations that might arise from an individual’s subjective costs or benefits of participation in an experiment.

Camerer (1995) notes9 that Harrison’s Allais-paradox experiment provides the only evidence that real, as opposed to hypothetical payoffs, influence behaviour in directions consistent with EUMH. He cites Camerer (1989) and Battalio, Kagel, and Jiranyakul (1990) who investigated the impact of financial incentives in Allais experiments, without finding much qualitative difference in behaviour between hypothetical and real lotteries.

Given the apparent uniqueness of Harrison’s contribution, Burke et al. (1996) set out to test whether viola-

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9. “…Harrison’s Allais-paradox experiments provide the only evidence that actually playing a gamble substantially reduces the rate of EU violations.” (Camerer, 1995, p. 634).
tions of expected utility are significantly diminished in Allais-type set-ups when payoffs are real rather than hypothetical. The upshot of their work is that:

“We share Harrison’s (1994) concern that the empirical case against expected utility theory might be overstated. Lottery-choice experiments often fail to satisfy the precepts of saliency and/or dominance. This is necessarily true when lottery choices are hypothetical.” (Burke et al., 1996; p. 623-624).

“We share Harrison’s (1994) concern that the empirical case against expected utility theory might be overstated. Lottery-choice experiments often fail to satisfy the precepts of saliency and/or dominance. This is necessarily true when lottery choices are hypothetical.” (Burke et al., 1996; p. 623–624).

**The Discovered Preference Hypothesis**

Another line of deference of comes in the form of the *discovered preference hypothesis* (DPH). The idea behind this hypothesis is that individuals have true underlying preferences that are EUMH consistent, but which experiments fail to reveal for a variety of reasons. If this is the case then, as Cubitt, Starmer, and Sugden (2001; p. 385) note, ‘the discovered preference hypothesis may seem to insulate expected utility theory from disconfirming experimental evidence’.

Consequently, Cubitt et al. (2001) identify the confounding effects to be expected in subject responses when DPH is true, and then consider how they might be controlled for. They argue for a design in which each subject faces just one distinct choice task, and for real. The authors then review the results of some tests of EUMH in which this design has been employed and find that such experiments ‘reveal the same violations of the independence axiom as other studies’. Cubitt et al. (2001) therefore argue that does not justify scepticism about recoded rejections of EUMH.

Interestingly, Cubitt et al. (2001) regard as an open question the issue of whether violations of EU become less frequent as subjects gain experience. This question is addressed by van de Kuilen and Wakker (2006) who aim to show that ‘learning can reduce violations of expected utility’. On the basis of a study which allows learning in an Allais paradox context, they find that:

“… choices converge to expected utility maximization if subjects are given the opportunity to learn by both thought and experience …” (van de Kuilen & Wakker, 2006; p. 155).

**Modified Probability Similarity**

Blavatskyy (2013c) notes an important feature of the Allais experiment which is an apparent *similarity* of probabilities in the second Allais question. Recall in the second choice situation:

\[ a_2 = (c_1, c_2, c_3; p_{1|2} = 0.10, p_{2|2} = 0, p_{3|2} = 0) \]

\[ a_2' = (c_1, c_2, c_3; p_{1|2} = 0.11, p_{2|2} = 0.89) \]

So, in the second decision context, individuals face a trade-off between the intermediate consequence \( c_2 \) achieved with probability 0.11 if they take action \( a_2 \) and the most desirable consequence \( c_1 \) which they will arrive at with probability 0.10, if they select action \( a_2' \). The argument is that since probability 0.11 \( \equiv \) probability 0.10, this similarity in probabilities may contribute to the Allais paradox outcome by inducing a kind of ‘optical illusion’ in experimental subjects. That opens the possibility of the Allais paradox disappearing in experiments which do not employ such a similarity of probabilities.

Blavatskyy (2013c) points to studies which modify the probability similarity feature of the original Allais design and find amelioration – and in some cases a reversal – of the Allais paradox as a result\(^ {10} \). He suggests, given the experimental situation surveyed by him that:

“… the existing experimental evidence tentatively suggests that the Allais paradox may be reversed in a design that decreases the similarity of probabilities in the second Allais question.” (Blavatskyy, 2013c; p. 61).

Blavatskyy (2013c) presents results from a new experiment\(^ {11} \) to support his conjecture that it is probability similarity that leads to Allais paradox behaviour. His experiment does indeed reveal interesting modifications of the Allais paradox – and indeed some cases of paradox
reversal. However, from the point of view of the bigger picture of the consistency of agent behaviour with the EUMH, his findings are that:

“Our experimental results reveal systematic violations not only of expected utility theory but also of a more general class of non-expected utility theories based on the betweenness axiom.” (Blavatskyy, 2013; p. 63).

**Some Field Evidence**

Huck and Müller (2012) take testing of the EUMH out of the laboratory and into general real world populations. They summarize their findings in doing this as follows:

“Earlier laboratory experiments (see Camerer, 1995 or Starmer, 2000 for surveys) have documented the Allais paradox in student samples. Our paper highlights that, if anything, these studies underestimate the true prevalence of the paradox in general populations ...” (Huck and Müller, 2012; p. 277, emphasis added).

Interestingly, and also as a result of *artefactual field experiments*¹², using data generated by very poor (rural) individuals in Ethiopia, India, and Uganda, Harrison et al. (2010) report roughly equal support for two major models of choice under risk and uncertainty, namely Expected Utility Theory and Prospect Theory (discussed below).

“...there is roughly equal support for the two major models of choice under uncertainty considered here. It is not the case that EUT or PT wins but that the data is consistent with each playing a roughly equal role, as in Harrison and Rutström (2009) for comparable laboratory tasks with university students.” (Harrison et al., 2010; p. 101).

**The Allais Paradox with Non-monetary Payoffs**

As Oliver (2003) notes, there have been numerous tests of EUMH using monetary outcomes. It is therefore interesting to round out this Section by noting a test of the Allais paradox in the context of gambles over health outcomes. Using a sample of 38 subjects, he found that the choice behaviour of 17 subjects strictly violated the independence axiom. He concluded that ‘the violations were thus significant and systematic’. He concludes that the evidence found in his investigation:

“... offers a further challenge to the descriptive validity of EUMH”. (Oliver, 2003; p. 35).

In light of the Allais paradox, the empirical work it has stimulated indicates that there are interesting circumstances in which EUMH is not a generally adequate theory of choice under risk; it is of considerable interest to review some of the alternative theories that have been formulated to account for choice under risk.

**CEOs and Expected Utility Maximization**

List and Mason (2011) begin by noting that the issue of whether individuals are expected utility maximizers is of more than academic interest because at stake in the answer are the underpinnings of a large number of models of choice under risk and uncertainty. List and Mason (2011) also note that the almost ubiquitous use of benefit-cost analysis in government agencies ‘renders the expected utility maximization paradigm literally the only game in town’. In order to investigate the empirical usefulness of EUMH, they study CEO preferences over small probability, high loss lotteries and using undergraduate students as our experimental control group, find:

“... that both our CEO and student subject pools exhibit frequent and large departures from expected utility theory. In addition, as the extreme payoffs become more likely CEOs exhibit greater aversion to risk.” (List & Mason, 2011; p. 114)

This is a particularly interesting study given the model of decision-making with ‘catastrophic risks’ that is considered later.

**MODELS ALTERNATIVE TO EU FOR ≲₄**

The empirical realization that the expected utility maximization hypothesis is not a perfectly general theory about how human beings go about making choices among risky actions lead to a flood of alternatives vying to either replace completely – or to plug some of its significant gaps. At a general level, it is clear what is needed. Since an expected utility function is of the form $EU(a) = \sum_{i=1}^{n} p_i u_B(c_i) + ... + p_n u_B(c_n)$ playing around with the way the probabilities enter this expression and/or the nature of the utility function used to evaluate consequences is the obvious strategy if one wants to augment and generalize EUMH.

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¹² Harrison et al. (2010; p. 80) call field experiments *artefactual* if they involve taking procedures from the laboratory and applying them in the field.
Approaches which have done one or both these things include: Allais’ theory of random choice (see Allais, 1988; Anticipated or Rank dependent utility (see Quiggin, 1982; Quiggin & Wakker, 1994); Cumulative Prospect Theory (see Tversky & Kahneman, 1992); Disappointment aversion theory (see, Gul, 1991; Grant & Kajii, 1998); Implicit (or linear) expected utility theory (see Yu, 2008); Implicit rank linear utility theory (see Luce & Fishburn, 1991; Carlier, 2008); Implicit weighted utility theory (see Cheung, 1992 for theory and Hu et al., 2012 for an application to transport choice); Generalised expected utility theory (see Machina, 1989; Quiggin, 1995; Chu and Halpern, 2008 for theory and Sopher and Giigliotti, 1997 for some tests); Prospect theory (see Hu, 2001; pp. 180-82): Suppose that a person is diagnosed with an illness that causes migraine at a frequency of 8 days per month. They are offered a treatment \( a = (c_1 = \text{pain free}; c_2 = 6 \text{ days of migraine}; c_3 = \text{no change}; p_{1|a} = 0.1, p_{2|a} = 0.7, p_{3|a} = 0.2) \).

To work out the \( RDU(a) \), the following steps need to be gone through.

1. Rank-order the consequences from least to most preferred. For sake of illustration, suppose: (no change) \( \leq (6 \text{ days of migraine}) \leq (\text{pain free}), \) where \( \leq \) means ‘at best as good as’.
2. Calculate both the cumulative probability of reaching each consequence or any other more preferred outcome \( \leq c \) and the cumulative probability of reaching all other more preferred consequences. Here, the probability of achieving at least no improvement = 1, the probability of achieving all other outcomes better than no improvement = 0.8; the probability of enjoying at least 6 days of migraine per month = 0.8; the probability of gaining all other outcomes better than 6 days of migraine per month 0.1; the probability of achieving at least complete recovery = 0.1; the probability of achieving all outcomes better than complete recovery = 0.
3. Transform by a weighting function the cumulative probabilities calculated in Step 2, where \( w \) is the one that behaves as follows: \( 1 \rightarrow w(1) = 1, 0.8 \rightarrow w(0.8), 0.1 \rightarrow w(0.1) \) and \( 0 \rightarrow w(0) = 0 \).
4. The decision weight \( \phi \) associated with each consequence is calculated by finding the difference between the transformed cumulative probability of achieving at least the outcome and the transformed cumulative probability of achieving all other more preferred consequences. Then \( \phi (\text{no change}) = 1 - w(0.8) \); \( \phi (6 \text{ days migraine}) = w(0.8) - w(0.1) \) and \( \phi (\text{pain free}) = w(0.1) \).
5. Then \( RDU(a) = [1 - w(0.8)]u_b(c_1) + w(0.8) - w(0.1)]u_b(c_2) + w(0.1)u_b(c_3) \).
Comparing the expression $EU(a) = \sum_{i=1}^{n} p_i u_B(c_i)$ with the expression for the rank-dependent expected utility of an action $RDU(a) = \sum_{i=1}^{n} \phi_i u_B(c_i)$ with $\phi = w(\sum_{i \in C} p_i) - w(\sum_{i \not \in C} p_i)$, it is clear how decision weights get substituted for probabilities.

While empirical support for RDEU has been found (see, for instance, Quiggin, (1993) – perhaps the most important contribution is the idea of decision weights, that were incorporated into the (Cumulative) Prospect Theory. The Cumulative Prospect Theory of Tversky and Kahneman (1992) discussed in the next Section is more general than RDEU since it allows probability weighting functions to differ in gains and in losses.

**Prospect Theory**

As Edwards (1996) notes, Prospect Theory was proposed by Kahneman and Tversky (1979) in response to their observation that the choices made by individuals in risky situations have several characteristics that are inconsistent with EUMH. The first is that individuals underweigh probable outcomes in comparison with outcomes that are certain – a phenomenon they named as the **certainty effect**. They also observed that this effect can lead to risk-aversion in choices involving certain gains and risk-seeking in choices involving certain losses – for details, see Kahneman and Tversky (1979 p.265). Secondly, individuals facing choices among different prospects disregard components that are common to all prospects under consideration – a phenomenon they termed as the **isolation effect**. This effect can cause the framing of a prospect to change choices (see, Kahneman & Tversky, 1979; p.271 for details). Thirdly, individuals display a reflection effect in which choices involving negative prospects and positive prospects are treated equivalently (see, Kahneman & Tversky, 1979, p.268 for details).

To account for these phenomena, Kahneman and Tversky (1979) supposed that in contrast to the EUMH where choices over risky actions are driven by considerations of final wealth and probabilities, decisions are made based on values assigned to gains and losses with respect to a reference point and decision weights. Schmidt and Zank (2012) rate reference dependence as the central innovation in Prospect Theory. They observe that it is consistent with at least three things: observed sign dependence (i.e. attitudes to risk and uncertainty depend on the sign of outcomes); diminishing sensitivity for outcomes (i.e. people are more sensitive to outcome changes near the reference point than to changes a long way from it) which shows up in an S – shaped utility which is convex over losses and concave over gains; and loss aversion, that is, a negative deviation from the reference point has a higher impact on an individual than does a positive deviation of equal size.


There is a finite state space, $S$, consisting of the states of the world $s_i$ for $i = 1, ..., n$ and a set of consequences $C$ given by an interval of the real line (to be thought of as wealth levels). In decisions under risk, each state $s_i$ has an objective probability $p_i$ of occurring with $\sum_{i=1}^{n} p_i = 1$. $A$ is the set of all actions and a particular action $a \in A$ is a function from $S$ to $A$ so that an action $a$ specifies for each state of the world $s_i$ the resulting consequence $a(s_i) \in C$. As in other versions of prospect theory, preferences over actions are reference dependent; so:

For any three acts, $a, a', a'' \in A$ to say that $a$ is weakly preferred to $a'$ when viewed from $a''$, where $a''$ is the reference act (the status quo position, say), we write $a' \preceq a'' - a$. Given this idea, it is possible to define a relative value function.

**Definition 5** (Relative value function): The desirability of the consequence of action $a$ in state $s$, relative to the conse-
Assumption 3P.1 The function \( v(a(s), a''(s)) \) is strictly increasing in its first argument.

Assumption 3P.2 \( v(a(s), a''(s)) = 0 \) when \( a(s) = a''(s) \).

We now define the idea of a function that assigns a real number to any act \( a \in A \) viewed from any reference action \( a'' \). Such a function will be called an expected relative value function.

**Definition 5** (Expected relative value): A function \( V : A \times A \rightarrow \mathbb{R} \) is an expected relative value function if it is defined by

\[
V(a, a') = \sum_{s} v(a(s), a''(s)) \times p(s)
\]

**Remark:** The function \( V(a, a') \) represents the preference order \( \preceq_{pr} \) in the sense for all \( a, a', a'' \in A \) we have \( a' \preceq_{pr} a \Leftrightarrow V(a', a'') \leq V(a, a') \).

As Schmidt et al. (2008) note:

“This approach is based on a state-contingent conception of reference-dependence. That is, gain/loss comparisons are made separately for each state of the world; thus, the pattern of gains and losses associated with any act \([a]\) viewed from any reference action \([a'']\), depends on the state-contingent juxtaposition of consequences in \([a]\) and \([a'']\).

(Schmidt et al., 2008, p. 207.)

Notice that the function \( V(a, a') = \sum v(a(s), a''(s)) \times p(s) \) is linear in probabilities. The third generation Prospect Theory relaxes that restriction by generalizing this form of \( V(a, a') \), using decision weights to \( V(a, a') = \sum v(a(s), a''(s)) \times w(s; a, a'') \) where \( w(s; a, a'') \) is the decision weight assigned to state \( s \) when \( a \) is being evaluated from the reference point \( a'' \).

Assumption 3P.3 The decision weights of an agent are determined cumulatively using a rank-dependent transformation of the sort introduced by Quiggin (1982) and discussed above.

In order to construct cumulative decision weights for a given pair of actions \( a, a'' \), states must be ordered according to the attractiveness of \( a' \)'s consequences in each state. The position of each state \( s \) is determined by the values of \( v(a(s), a''(s)) \). Consequently, as Schmidt et al. (2008; p. 208) argue, the ordering of states must be constructed separately for each \( a, a'' \) pair. Also, in the spirit of cumulative prospect theory, there must be separate rank-dependent transformations of probability for gains and losses that are mirror-images of one another.

Illustration (Schmidt et al., 2008; p.208-209): Consider any pair \((a, a'') \in A \times A\). Relative to that pair, there is a weak gain in a state \( s \) if \( a''(s) \leq a(s) \) and a strict loss if \( a''(s) > a(s) \). Partition the total number of states \( n \) into \( m^+ = \frac{n}{2} \) the number of states in which there are weak gains and \( m^- = n - m^+ \) the number of states in which there are strict losses. Subscripts on states are assigned so that, \( \forall i, j = 1, ..., n, i \) comes after \( j \) (so \( j < i \)), if and only if it is the case that \( v(a(s_i), a''(s_j)) \leq v(a(s_j), a''(s_i)) \). Then, states with weak gains are indexed by \( m^+, ..., 1 \) while the states with strict losses are indexed by \(-1, ..., -m^-\). Cumulative decision weights can be defined by letting:

\[
w(s; a, a'') = \begin{cases} 
  w^+(p_i) & \text{if } i = m^+ \\
  w^+(\sum_{i=m^+}p_i) - w^+(\sum_{i=1}p_j) & \text{if } 1 \leq i \leq m^+ - 1 \\
  w^-(\sum_{i=1}p_j) - w^-\left(\sum_{i=m^-}p_i\right) & \text{if } m^- + 1 \leq i \leq -1 \\
  w^-(p_i) & \text{if } i = m^- 
\end{cases}
\]

Here both \( w^+ \) and \( w^- \) are strictly increasing maps of \([0, 1]\) onto \([0, 1]\). They are interpreted, respectively as the probability weighting functions for the gain and loss domains. The third generation Prospect Theory is closed by the following:

Assumption 3P.4 When making choices in risky environments, agents behave so as to maximize \( V(a, a') = \sum v(a(s), a''(s)) \times w(s; a, a'') \).

Special cases: 1. RDEU is the special case in which decision weights are untransformed state probabilities so that \( w^+(p_i) = w^-(p_i) = p_i \); 2. Cumulative prospect theory emerges when the relative value function has the form \( v(a(s), a''(s)) = u(a(s), a''(s)) \) where \( u(\cdot) \) is some value function and where the reference actions are certainties so that \( u''(s) = a''(s) \), \( \forall i, j = 1, ..., n \); 3. Expected utility theory is the special case in which decision weights are untransformed state probabilities and relative value is independent of the reference outcome, so \( w(s; a, a'') = p_i \) and \( v(a(s), a''(s)) = EU(a(s)) \) where \( EU(\cdot) \) is an expected utility function.
In summary, the third generation Prospect Theory as presented by Schmidt et al. (2008) retains all the empirically inspired features of the earlier versions of Prospect Theory such as loss aversion, diminishing sensitivity, and non-linear probability weightings, and adds in the generalization of allowing reference points to be uncertain.

How well does it perform empirically? According to Schmidt et al. (2008), the third generation Prospect Theory: “… retains all the predictive power of those previous variants, but in addition provides a framework for determining the money valuation that an agent places on a lottery … More surprisingly, when [the theory] is made operational by using simple functional forms with parameter values derived from existing experimental evidence, it predicts observed patterns of preference reversal across a wide range of specifications … consistent with the range in which that phenomenon has in fact been observed. … The result is a flexible and parsimonious model of choice under uncertainty which organises a large body of experimental evidence.” (Schmidt et al., 2008; pp. 220-221).

In a particularly interesting study of the behaviour of finance professionals, Abdellaoui et al. (2013) begin by noting that much of the empirical support for Prospect Theory, of the sort reported in Schmidt et al. (2008), for example, comes from laboratory experiments using student subjects. The authors are interested to know if this empirical support extends to the ‘real world’ – in particular to the behaviour of hard-nosed finance professionals.

To that end, Abdellaoui, Bleichrodt, and Kammoun (2013) assess the descriptive potency of Prospect Theory using a sample of private bankers and fund managers and in the event find ‘clear support for Prospect Theory. In particular the authors found that:

“Our financial professionals behaved according to prospect theory and violated expected utility maximization. They were risk averse for gains and risk seeking for losses and their utility was concave for gains and (slightly) convex for losses. They were also averse to losses, but less so than commonly observed in laboratory studies … A substantial minority focused on gains and largely ignored losses, behavior reminiscent of what caused the current financial crisis.” (Abdellaoui et al., 2013; p.411).

This is an interesting finding, especially when read in conjunction with the self-reports of finance professionals in the Colloquium Section of this issue.

**Catastrophic Risk and the Topology of Fear**

In a series of papers, Chichilnisky (1996, 2000, 2002, 2009, 2010), revisited the von Neumann – Morgenstern axioms that underpin EUMH, in particular, the continuity axiom for preferences over risky alternatives. Its implication, discussed earlier, that preference rankings are insensitive to small changes in probabilities, even for catastrophic outcomes, is the focus here. Chichilnisky’s motivation is nicely captured in the following remark:

“For many years experimental observations have raised questions about the rationality of economic agents – for example, the Allais Paradox or the Equity Premium Puzzle. The problem is a narrow notion of rationality that disregards fear [and rare but catastrophic events]. This article extends the notion of rationality with new axioms of choice under uncertainty and the decision criteria they imply … In the absence of catastrophes, the old and the new approach coincide, and both lead to standard expected utility. A sharp difference emerges when facing rare events with important consequences, or catastrophes.” (Chichilnisky, 2009, p. 807).

Drawing on results in experimental psychology showing that human brains behave quite differently in potentially catastrophic situations that invoke extreme fear, to the way they operate ‘normally’13, Chichilnisky (2009) argues that these experimental observations would seem ‘to be relevant to the issue of rationality in decision-making under uncertainty’ – and to the general applicability of EUMH.

The particular shortcoming of an expected utility representation of preferences over risky alternatives that Chichilnisky (1996, 2000, 2002, 2009, 2010) focuses on, is that it underestimates (reasonable and rational) responses to rare events no matter how catastrophic their consequences may be.

She argues that:

“… the insensitivity of expected utility to rare events and the attendant inability to explain responses to events that invoke fear, are the source of many of the experimental

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paradoxes and failures of that have been found over the years.” (Chichilnisky, 2009, p. 808).

The observation from which Chichilnisky’s work departs is contained in the following example due to Arrow (1971, pp. 48–49), an example identical in spirit to the example in Mas-Colell et al. (1995; p. 171) considered by us earlier.

Arrow’s Example: Let \( a_1 \) be an action that involves receiving one cent, \( a_2 \) an action that involves receiving zero cents, and \( a_3 \) an action involving receiving one cent and facing a small probability of death. Suppose that \( u_B \) (zero cents) < \( u_B \) (one cent), then the continuity axiom underlying the EUMH (Axiom 4 above), requires that the third action involving death and one cent should be preferred to the second involving zero cents when the probability of death is small enough (but still positive).

Supposing that death is regarded by an individual as a catastrophe, then the insensitivity to rare events forced on decision-makers by the EUMH is a potential source of failure of the hypothesis – and one that Chichilnisky (1996, 2000, 2009) addresses by replacing the continuity requirement in Axiom 4 above with a condition requiring sensitivity to rare events. The details of how all this works are quite technical; however, the gist is as follows. Replace the Axioms underlying Proposition 1 above with the following:

**Axiom 1’**: The ranking functional \( FR: A \to \mathbb{R} \) is linear and continuous on the space of risky actions

**Axiom 2’**: The ranking functional \( FR: A \to \mathbb{R} \) is sensitive to rare events

**Axiom 3’**: The ranking functional \( FR: A \to \mathbb{R} \) is sensitive to frequent events.

**Example** (Chichilnisky, 2009; p. 811): An example of behaviour that is consistent with these three axioms is, when a portfolio is chosen, it is structured so as to maximize expected utility while seeking to minimize total value losses in the event of a catastrophe.

In a set-up governed by these axioms, the following can be shown to be generally true:

(Chichilnisky, 2009; Theorem 1]: In the absence of rare events, a ranking of the space of risky actions that satisfies Axioms 1’, 2’, and 3’ is consistent with Expected Utility Theory.

One reason for being interested in the approach of Chichilnisky is that it is consistent with the behaviour of agents who face ‘heavy tails’ in the distributions of, say, returns in financial markets. As she points out:

“... [this] new understanding of rationality consistent with previously unexplained observations about decisions involving rare and catastrophic events, decisions involving fear, the Equity Premium Puzzle, ‘jump diffusion’ processes and ‘heavy tails’ ...” (Chichilnisky, 2009; p. 807).

This is an interesting collection of ideas at a theoretical level, but does this approach have any empirical support? Chanel and Chichilnsiky (2009) addressed this question experimentally – in a set-up the details of which are described in Chanel and Chichilnsiky (2009; pp. 274–78). In brief, they study decisions made under conditions of fear, when a catastrophic outcome is introduced in a risky alternative or lottery. The experimental results reported by Chanel and Chichilnsiky (2009):

“... provide evidence that fear influences the cognitive process of decision-making by leading some subjects to focus excessively on catastrophic events. Such heterogeneity in subjects’ behavior, while not consistent with EU-based functions, is fully consistent with the new type of utility function implied by the new axioms [i.e. Axioms 1’, 2’, and 3’].”

For studies specifically focused on the effects of fear in financial markets see Lo et al. (2005), Chang and Tu (2011), Hu and McInish (2013).

**A Methodological Suggestion**

There is an interesting methodological suggestion in Harrison and Rutström (2009) to abandon the search for a unique theory to explain all choices under risk (and uncertainty). Harrison and Rutström (2009) suggest that we should aim instead for a combination of hypotheses, the weights on which correspond to the percentage of the population exhibiting expected utility behaviour, prospect theory behaviour and so on. As Harrison and Rutström (2009) put it:
“Debates over the validity of … models have often been framed as a horse race, with the winning theory being declared on the basis of some statistical test in which the theory is represented as a latent process explaining the data. In other words, we seem to pick the best theory by ‘majority rule’. If one theory explains more of the data than another theory, we declare it the better theory and discard the other one. In effect, after the race is over, we view the horse that ‘wins by a nose’ as if it was the only horse in the race. The problem with this approach is that it does not recognize the possibility that several behavioural latent processes may coexist in a population.” (Harrison & Rutström, 2009, p. 134).

As Harrison et al. (2010) put it:

“… substituting PT for EUT would be tantamount to replacing one ‘half wrong’ assumption with another. [Consequently] policies should not be designed under the assumption that one or other theory explains all behaviour. Policymakers are therefore faced with the substantial challenge of examining the sensitivity of the predicted impact of policy interventions to the replacement of single key assumptions with a probability distribution over competing assumptions …” (Harrison et al., 2010; p. 101-102).

This is an interesting suggestion – and one that has been tried in other areas of economics – such as macroeconomics (e.g., Hansen & Sargent, 2001). It remains to be seen the benefits it might yield in the current context.

**DECISION-MAKING UNDER UNCERTAINTY**

Leaving the relatively safe harbour of decision-making under risk – where agents at least have access to objective probabilities over states of the world – the paper now heads into the ocean of choice under uncertainty where objective probabilities over states are not available (hopefully without having to jettison everything we have learned about choice in a risky environment). The reason for needing to make this adjustment in our thinking is that, outside the context of a casino, it is rare for agents to have access to objective probabilities about events and states of the world – a point emphasized by Keynes (1921) and Knight (1921). Decision-makers are therefore typically forced to make subjective judgements about the likelihood of various events and states in order to select an action.

Despite this significant change in context from risk to uncertainty, the central question that we are trying to answer remains essentially the same, namely: How are we to suppose agents order the space of actions available to them – and then make a selection? We now look at some possible answers to this question.

In order to get started and following Wakker (2008), we introduce the following ideas and notation. Decision-making under uncertainty involves choices between actions generically written as \((c_1; e_1; c_2; e_2; ...; c_n; e_n)\) yielding a monetary consequence \(c_i\) if event \(e_i\) happens with \(i = 1, ..., n\). The events \(e_1, e_2, ..., e_n\) are mutually exclusive and exhaustive and the agent does not know for sure whether they will occur – nor do they have any objective probabilities about the likelihood of any event happening. Because agents are unsure about which event will happen, they are also uncertain about which consequence they will enjoy/suffer as a result of taking an action. They are therefore making decisions under uncertainty.

**Subjective Expected Utility Theory**

An ‘obvious’ approach to getting a theory of choice under uncertainty is to use the machinery developed for choice under risk, but instead of employing objective probabilities, we suppose instead that agents are able to come up with subjective probabilities \(p_i\) for \(i = 1, ..., n\) over the space of events. Agents can then be imagined to order the space of uncertain actions using a subjective expected utility function. This is essentially the approach suggested by de Finetti (1931), Ramsey (1931), and Savage (1954).

One problem that this approach runs into is the so-called Ellsberg paradox – which plays a role in choice under uncertainty to that played by the Allais paradox in risky choice situations. As it is such an important empirical result, we spend a little time exploring it.

**Ellsberg Paradox**

Imagine a situation with two urns, \(K\) and \(A\). Urn \(K\) has a known composition of 50 red balls and 50 black balls, while the nature of urn \(A\) is ambiguous because it has 100 red and black balls in total, but in unknown proportion. One ball is drawn from each urn and let:

- \(r_K\) denote the event that a red ball is drawn from the known urn \(K\)
- \(b_K\) denote the event that a black ball is drawn from the known urn \(K\)
\( r_j \) denote the event that a red ball is drawn from the ambiguous urn \( A \)

\( b_A \) denote the event that a black ball is drawn from the known urn \( K \)

Experimental evidence shows that a common preference pattern is:

\[
(b_{A}; \$100; r_{A}; \$0) < (b_{K}; \$100; r_{K}; \$0) \text{ and } (b_{A}; \$0; r_{A}; \$100)<(b_{K}; \$0; r_{K}; \$100)
\]

Now, if people are revealing subjective probabilities by these preference rankings, then what they are saying is that \( p^*(b_{A}) < p^*(b_{K}) \) and \( p^*(r_{A}) < p^*(r_{K}) \). However, if these quantities are to be probabilities then \( p^*(b_{A}) + p^*(r_{A}) = 1 = p^*(b_{K}) + p^*(r_{K}) \). But \( p^*(b_{A}) + \epsilon_1 = p^*(b_{K}) \) and \( p^*(r_{A}) + \epsilon_2 = p^*(r_{K}) \); so, \( 1 = p^*(b_{A}) + p^*(r_{A}) = p^*(b_{A}) + \epsilon_1 + p^*(r_{A}) + \epsilon_2 = 1 + (\epsilon_1 + \epsilon_2) \), which is a contradiction. Consequently, the behaviour exhibited in the Ellsberg’s paradox violates the subjective expected utility hypothesis.

As Zhang (2002) notes, the Ellsberg study stimulated a great deal of empirical work which reinforced the early Ellsberg findings that the Subjective Expected Utility (SEU) model, axiomatized by Savage (1954), is not generally able to account for aversion to uncertainty or aversion to ambiguity. The leading contenders in this regard are Choquet utility, Wald’s (1950) multiple priors model and the Case Based Approach of Giboa and Schmieder (2001). As work on these topics is taken up by others in the Colloquium, we stop at this point and offer a brief conclusion.

CONCLUSION

The aim of this paper has been to present something of the microeconomics of uncertainty and financial decision making. This is a fascinating and lively area of Economics – particularly at points where it interacts with cognate disciplines such as Finance. While as Giboa (2009) recently remarked, no clean, unified and general theory of behaviour under risk and uncertainty is around the corner, our view is certainly clearer now than it has ever before been about the obstacles in the path of obtaining such a theory.

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